JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS

CLASS :- 12th **PAPER CODE :- CWT-4**

CHAPTER :- MATRICES & DETERMINANTE

ANSWER KEY

1.	(A)	**2.**	(A)	**3.**	(D)	**4.**	(D)	**5.**	(A)	**6.**	(C)	**7.**	(A)
8.	(C)	**9.**	(D)	**10.**	(C)	**11.**	(D)	**12.**	(B)	**13.**	(B)	**14.**	(D)
15.	(C)	**16.**	(C)	**17.**	(A)	**18.**	(C)	**19.**	(B)	**20.**	(C)	**21.**	2
22.	64	**23.**	36	**24.**	2	**25.**	50	**26.**	4	**27.**	0	**28.**	63
29.	4	**30.**	6										

SOLUTIONS

1. (A)

Sol. Given, in ΔABC $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$

$\Rightarrow 1(c^2 - ab) - a(c - a) + b(b - c) = 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$\Rightarrow (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc)$

$+ (c^2 + a^2 - 2ca) = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

Here, sum of squares of three members can be zero if and only if $a = b = c$

$\Rightarrow \Delta ABC$ is equilateral

$\Rightarrow \angle A = \angle B = \angle C = 60^o$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C =$

$(\sin^2 60^o + \sin^2 60^o + \sin^2 60^o)$

$= 3 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4}$.

2. (A)

Sol. $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$= (1 - \log_z y \log_y z) - \log_x y(\log_y x - \log_z x \log_y z)$

$+ \log_x z(\log_y x \log_z y - \log_z x)$

$= (1 - 1) - (1 - \log_x y \log_y x) + (\log_x z \log_z x - 1) = 0$

{Since $\log_x y . \log_y x = 1$}.

3. (D)

Sol. Let A be the first term and R be the common ratio of the G.P. then,

$l = AR^{p-1} \Rightarrow \log l = \log A + (p - 1)\log R$ …..(i)

$m = AR^{q-1} \Rightarrow \log m = \log A + (q - 1)\log R$ …..(ii)

$n = AR^{r-1} \Rightarrow \log n = \log A + (r - 1)\log R$ …..(iii)

Multiplying (i), (ii) and (iii) by $(q - r), (r - p)$ and $(p - q)$ respectively and adding we get,

$\log l(q - r) + \log m(r - p) + \log n(p - q) = 0$

$\therefore \Delta = 0$.

4. (D)

Sol. Given $x^a y^b = e^m, x^c y^d = e^n$

$\Rightarrow a \log x + b \log y = m$ and

$c \log x + d \log y = n$

By Cramer's rule, $\log x = \frac{\Delta_1}{\Delta_3}$ and

$\log y = \frac{\Delta_2}{\Delta_3}$

$\Rightarrow x = e^{\Delta_1/\Delta_3}$ and $y = e^{\Delta_2/\Delta_3}$.

5. (A)

Sol. $\Delta = -(a^3 + b^3 + c^3 - 3abc)$

$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= -\frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$,

which is clearly negative because of the given conditions.

6. (C)

Sol. The system of homogeneous equations

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

has a non-trivial solution (since x, y, z are not all zero)

If $\Delta = \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

i.e., if $(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

i.e., if $a^2 + b^2 + c^2 + 2abc = 1$.

7. (A)

Sol. If A is square matrix of order 3, then

$|-2A| = (-2)^3 |A| = -8|A|$.

8. (C)

Sol. As the system of equations has a non-trivial solution

$$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0, \text{ by } \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\Rightarrow a(b-1)(c-1) - 1.(1-a)(c-1)$

$-1.(1-a)(b-1) = 0$

$$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} - 1 + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1.$$

9. (D)

Sol. We know $A.adj(A) = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

$$\therefore |A|.|adj(A)| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$\therefore |A|.adj|A| = |A|^3$

Now question gives $|A| = 8$

$\therefore 8.adj|A| = 8^3$ or $adj|A| = 8^2 = (2^3)^2 = 2^6$.

10. (C)

Sol. Since

$$A^2 = A.A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}.$$

11. (D)

Sol. $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$

$$\Rightarrow A.A = A^2 = \begin{bmatrix} 6 & 11 & 7 \\ -11 & 4 & -11 \\ 7 & 11 & 12 \end{bmatrix},$$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then,

$$A^2 + 9I = \begin{bmatrix} 15 & 11 & 7 \\ -11 & 13 & -11 \\ 7 & 11 & 21 \end{bmatrix}.$$

12. (B)

Sol. $|A| = 1 + \tan^2\frac{\theta}{2} = \sec^2\frac{\theta}{2}$

$AB = I \Rightarrow B = IA^{-1}$

$$\frac{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}}{\sec^2\frac{\theta}{2}} = \cos^2\frac{\theta}{2}.A^T.$$

13. (B)

Sol. $(A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$.

14. (D)

Sol. $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix}$$

$$A.A^2 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix}$$

$\Rightarrow A^3 - 3A^2 - A + 9I_3 = 0$.

15. (C)

Sol. $\begin{matrix} 3X + 2Y = I \\ 2X - Y = O \end{matrix} \Rightarrow \begin{matrix} 3X + 2Y = I \\ 4X - 2Y = O \end{matrix} \Rightarrow \begin{matrix} 7X = I \\ X = \frac{1}{7}I \end{matrix}$

(Solving simultaneously)

Therefore from (i),

$2Y = I - \frac{3}{7}I = \frac{4}{7}I \Rightarrow Y = \frac{2}{7}I$.

16. (C)

Sol. It is obvious.

17. (A)

Sol. $A_{3\times4} \Rightarrow A'_{4\times3}$; Now $A'B$ defined $\Rightarrow$ B is $3\times p$

Again $B_{3\times p}A'_{4\times3}$ defined $\Rightarrow$ $p = 4$

$\therefore$ B is 3×4.

18. (C)

Sol. $A' = [1\ 2\ 3]$ therefore

$$AA' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}[1\ 2\ 3] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}.$$

19. (B)

Sol. It is a fundamental concept.

20. (C)

Sol. Since

$$A^2 = A.A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix}$$

And $A^3 = \begin{bmatrix} a^3 & 0 & 0 \\ 0 & b^3 & 0 \\ 0 & 0 & c^3 \end{bmatrix}, ...$

$$\Rightarrow A^n = A^{n-1}.A = \begin{bmatrix} a^{n-1} & 0 & 0 \\ 0 & b^{n-1} & 0 \\ 0 & 0 & c^{n-1} \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}.$$

Note: Students should remember this question as a formula.

21. 2

Sol. $A^2 = I \Rightarrow A^4 = A^6 = ... = I$

Now $A^x = I \Rightarrow x = 2, 4, 6, 8, ...$

$\sum_x (\cos^x\theta + \sin^x\theta) = \cos^2\theta + \cos^4\theta + ... + \sin^2\theta + \sin^4\theta + \sin^6\theta + ...$

$= \dfrac{\cos^2\theta}{1-\cos^2\theta} + \dfrac{\sin^2\theta}{1-\sin^2\theta} = \cot^2\theta + \tan^2\theta$

which has minimum value 2.

22. 64

Sol. A diagonal matrix is commutatives with every square matrix if it is scalar matrix. So every diagonal element is 4.

so $|A| = 64$

23. 36

Sol. given $| = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \Rightarrow abc = 120 = 2^3 \times 3^1 \times 5^1$

Case - I All are Positive $= {}^5C_2 \times {}^3C_2 \times {}^3C_2 = 90$

Case - I one Positive and two negative

$= 3 \times ({}^5C_2 \times {}^3C_2 \times {}^3C_2) = 270$

So number of possible matrices = 90 + 270 = 360

24. 2

Sol. Let Δ

$$= \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Operate : $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+a) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} = 2(a+b+c)\begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix}$$

Operate : $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$\Rightarrow D = 2(a+b+c)\begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix}$$

$$= 2(a+b+c)\ 1. \begin{vmatrix} b-c & c-a \\ b-a & c-b \end{vmatrix}$$

open w.r.t. R_1

$= 2(a + b + c) [(b - c)(c - b) - (c - a)(b - a)] = 2(a + b + c) [bc - b^2 - c^2 + cb - (cb - ac - ab + a^2)]$

$= 2(a + b + c)(ab + bc + ca - a^2 - b^2 - c^2)$

$\leq 0 \quad \Rightarrow \quad a = b = c$

25. 50

Sol. $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

$\Rightarrow \dfrac{1}{a_1}, \dfrac{1}{a_2}, ..., \dfrac{1}{5}, \dfrac{1}{4}, ..., \dfrac{1}{a_9}$ are in A.P.

$d = \dfrac{1}{4} - \dfrac{1}{5} = \dfrac{1}{20}$

$\Rightarrow \dfrac{1}{5} = \dfrac{1}{a} + \dfrac{3}{20} \Rightarrow \dfrac{1}{a_1} = \dfrac{1}{20}$

$\therefore \dfrac{1}{a_n} = \dfrac{1}{a_1} + \dfrac{(n-1)}{20} = \dfrac{n}{20}$

$\Rightarrow a_n = \dfrac{20}{n}$

Hence, **D**

$$= \begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix} = \frac{(20)^3}{4\times 7}\begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{4}{5} & \frac{2}{3} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \frac{(20)^3}{4\times 7}\begin{vmatrix} 0 & \frac{-3}{10} & \frac{-1}{3} \\ 0 & \frac{-3}{40} & \frac{-1}{9} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix} = \frac{50}{21} \Rightarrow$$

21D = 50

26. 4

Sol. LHS

$$= \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

Operate: $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Operate : $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 0 & (a+b+c) & 0 \\ 0 & 0 & (c+a+b) \end{vmatrix}$$

Expand by C_1

$= 2(a+b+c).1. \begin{vmatrix} a+b+c & 0 \\ 0 & a+b+c \end{vmatrix} =$

$2(a+b+c)^3 \Rightarrow k = 2$ and $\alpha = \beta = \gamma = 1$

27. 0

Sol. $A'A = I$

$|A - I| = |A - A'A|$

$\Rightarrow \quad |A - I| = |A|\,|I - A'|$

$\Rightarrow \quad |A - I| = -1 . |A' - I|$

$\Rightarrow \quad |A - I| = -|A - I|$

$\Rightarrow \quad |A - I| = 0$

28. 63

Sol. $A^2 = A \Rightarrow \quad A^{-1} A^2 = A^{-1}A$

$\Rightarrow \quad A = I$

$\therefore (I + A)^6 = (I + I)^6 = (2I)^6 = 64\,I = I + KI = (K + 1)\,I \quad \therefore K + 1 = 64 \Rightarrow K = 63$

29. 4

Sol.

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

$$\Rightarrow \frac{1}{abc} \begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

$$\Rightarrow \frac{abc}{abc} \begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$(ab+bc+ca) \begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\Rightarrow (ab+bc+ca) \begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & 0 \\ ac+bc & 0 & -(ab+bc+ca) \end{vmatrix}$$

$= (ab + bc + ca)^3$

30. 6

Sol.

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$= (2 + 4\sin 2x)$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (2+4\sin 2x) \begin{vmatrix} 1 & \cos^2 x & 4\sin 2x \\ 1 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$$f(x) = (2+4\sin 2x) \begin{vmatrix} 1 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$\Rightarrow \quad f(x)_{max} = 6$