JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 12th **PAPER CODE :- CWT-3**
CHAPTER :- INVERSE TRIGONOMETRIC FUNCTIONS

ANSWER KEY

1.	(A)	**2.**	(B)	**3.**	(C)	**4.**	(D)	**5.**	(D)	**6.**	(D)	**7.**	(B)
8.	(B)	**9.**	(D)	**10.**	(B)	**11.**	(C)	**12.**	(B)	**13.**	(A)	**14.**	(B)
15.	(C)	**16.**	(D)	**17.**	(C)	**18.**	(B)	**19.**	(B)	**20.**	(B)	**21.**	3
22.	5	**23.**	2	**24.**	6	**25.**	12	**26.**	38	**27.**	6	**28.**	2
29.	2	**30.**	5										

SOLUTIONS

1. (A)

Sol. $x^2-\sqrt{2}x+\sqrt{3-2\sqrt{2}}=0$

$\Rightarrow x^2-\sqrt{2}\,x+\sqrt{2}-1=0$

$\Rightarrow x^2-1-\sqrt{2}\,(x-1)=0$

$\Rightarrow (x-1)\left(x+1-\sqrt{2}\right)=0$

$x=1,\ \sqrt{2}-1$

$\therefore\ \alpha=1$ and $\beta=\sqrt{2}-1$.

Hence, $\cos^{-1}\alpha+\tan^{-1}\alpha+\tan^{-1}\beta=0+\frac{\pi}{4}+\frac{\pi}{8}$

$=\frac{3\pi}{8}$. **Ans.**]

2. (B)

Sol. $\forall\, x\in R,\ \tan^{-1}x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$. So $\cos(\tan^{-1}x)\in(0,1]$. **Ans.**

3. (C)

Sol. $\sum_{r=2}^{\infty}\tan^{-1}\left(\frac{(r-2)-(r-3)}{1+(r-3)(r-2)}\right)$

$=\sum_{r=2}^{\infty}\left(\tan^{-1}(r-2)-\tan^{-1}(r-3)\right)$

$=\tan^{-1}0-\tan^{-1}(-1)$
$\tan^{-1}1-\tan^{-1}0$
$\tan^{-1}2-\tan^{-1}1$
:
:
$\tan^{-1}(n-2)-\tan^{-1}(n-3)$

$S_n=\tan^{-1}(n-2)+\frac{\pi}{4}$

$\therefore S_\infty=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$. **Ans**

4. (D)

Sol. $f(x)=\sin^{-1}(\sin x)=\begin{cases}x, & \frac{-\pi}{2}\le x\le\frac{\pi}{2}\\ \pi-x, & \frac{\pi}{2}<x\le\frac{3\pi}{2}\\ x-2\pi, & \frac{3\pi}{2}<x\le\frac{5\pi}{2}\end{cases}$

$g(x)=\cos^{-1}(\cos x)=\begin{cases}x, & 0\le x\le\pi\\ 2\pi-x, & \pi<x\le 2\pi\end{cases}$

Now, verify alternatives.]

5. (D)

Sol. $x\tan^{-1}c+y\left(\frac{\pi}{2}-\tan^{-1}c\right)+2=0$

$\Rightarrow (x-y)\tan^{-1}c+\left(\frac{\pi y}{2}+2\right)=0$

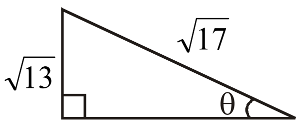
$\therefore$ The fixed point is $\left(\frac{-4}{\pi},\frac{-4}{\pi}\right)$. **Ans.**]

6. (D)

Sol. Given that, $\cot\left(\sin^{-1}\sqrt{\frac{13}{17}}\right)=\sin(\tan^{-1}x)$

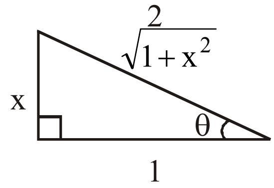
Note that x must be positive.

Put $\sin^{-1}\sqrt{\frac{13}{17}}=\theta$.



$\therefore$ L.H.S. $=\frac{2}{\sqrt{13}}$

Put $\tan^{-1}x=\phi$



$\therefore$ R.H.S. $=\frac{x}{\sqrt{1+x^2}}$

So, given equation is $\frac{2}{\sqrt{13}}=\frac{x}{\sqrt{1+x^2}}$ (on squaring)

$\Rightarrow\ x=\frac{2}{3}$ (As $x>0$). **Ans.**]

7. (B)

Sol. $\theta = \tan^{-1}\left(\dfrac{\sin\frac{\pi}{18}+\sin\frac{2\pi}{18}}{\cos\frac{\pi}{18}+\cos\frac{2\pi}{18}}\right)$

$= \tan^{-1}\left(\dfrac{2\sin\frac{3\pi}{36}\cos\frac{\pi}{36}}{2\cos\frac{3\pi}{36}\cos\frac{\pi}{36}}\right)$

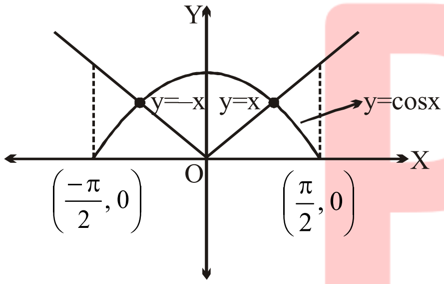
$= \tan^{-1}\left(\tan\dfrac{3\pi}{36}\right)$

$\theta = \dfrac{3\pi}{36}$ radian $= \dfrac{\pi}{12}$. Ans.

8. (B)

Sol. As, $\left|\sin^{-1}(\sin x)\right| = |x|$, for $x \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$

$\therefore$ From above graph, the equation



$\left|\sin^{-1}(\sin x)\right|$ = cos x has two solutions, in

$\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$. **Ans.**

9. (D)

Sol. Sum $= \displaystyle\sum_{r=1}^{\infty}\cot^{-1}\left(\frac{r^2}{2}+\frac{15}{8}\right)$

$= \displaystyle\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{\frac{r^2}{2}+\frac{15}{8}}\right) = \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{2}{r^2+\frac{15}{4}}\right)$

$= \displaystyle\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{2}{4+r^2-\frac{1}{4}}\right)$

$= \displaystyle\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{2}{4+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$

$= \displaystyle\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{\frac{1}{2}}{1+\left(\frac{r+\frac{1}{2}}{2}\right)\left(\frac{r-\frac{1}{2}}{2}\right)}\right)$

$= \displaystyle\sum_{r=1}^{\infty}\tan^{-1}\left\{\frac{\frac{\left(r+\frac{1}{2}\right)}{2}-\frac{\left(r-\frac{1}{2}\right)}{2}}{1+\frac{\left(r+\frac{1}{2}\right)}{2}\cdot\frac{\left(r-\frac{1}{2}\right)}{2}}\right\}$

$= \displaystyle\lim_{n\to\infty}\sum_{r=1}^{n}\left(\tan^{-1}\left(\frac{r+\frac{1}{2}}{2}\right)-\tan^{-1}\left(\frac{r-\frac{1}{2}}{2}\right)\right)$

$\displaystyle\lim_{n\to\infty}\left(\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{1}{4}\right)+\left(\tan^{-1}\frac{5}{4}-\tan^{-1}\frac{3}{4}\right)$

$= +\left(\tan^{-1}\dfrac{7}{4}-\tan^{-1}\dfrac{5}{4}\right)$

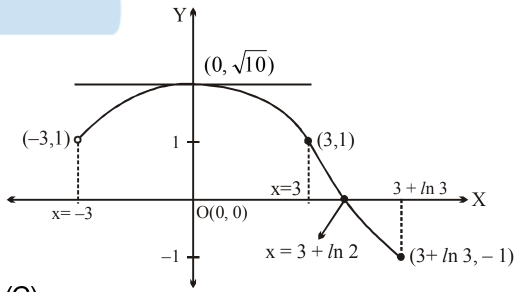
$\ldots\ldots\ldots\left(\tan^{-1}\left(\dfrac{n}{2}+\dfrac{1}{4}\right)-\tan^{-1}\left(\dfrac{n}{2}-\dfrac{1}{4}\right)\right)$

$= \displaystyle\lim_{n\to\infty}\left(\tan^{-1}\left(\frac{n}{2}+\frac{1}{4}\right)-\tan^{-1}\frac{1}{4}\right)$

$= \dfrac{\pi}{2}-\tan^{-1}\dfrac{1}{4} = \cot^{-1}\dfrac{1}{4} = \tan^{-1}4$. **Ans.**]

10. (B)

Sol.



11. (C)

Sol. X = x + 2011 and Y = y + 2012, (X & Y both can not be negative)

$\tan^{-1}\left(\dfrac{X+\frac{1}{Y}}{1-\frac{X}{Y}}\right) = \tan^{-1}2 \Rightarrow XY + 1 = 2(Y - X)$

$\Rightarrow Y = \dfrac{1+2X}{2-X}$

$\Rightarrow Y = \dfrac{5}{2-X} - 2 \Rightarrow$ (X, Y) = (1, 3) (3, – 7), (7, – 3) and (– 3, – 1)

$\therefore$ (x, y) ≡ (x = – 2010, y = 2009), (x = – 2008,. y = – 2019), (x = – 2004, y = – 2015).

But (x = – 2014, y = – 2013) rejected.

[For Y to be an integer, 2 – x = ± 1 or ± 5 ⇒ x = 1, 3, 7, – 3]. **Ans.**]

12. (B)

Sol. $\because \ \frac{-\pi}{2} \le \sin^{-1}x,\ \sin^{-1}y,\ \sin^{-1}z \le \frac{\pi}{2}$

$\Rightarrow \frac{-\pi^3}{8} \le (\sin^{-1}x)^3, (\sin^{-1}y)^3, (\sin^{-1}z)^3 \le \frac{\pi^3}{8}$

$\therefore$ Given equation holds good iff

$\sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

$\Rightarrow x = y = z = 1$

$\therefore 2x - 3y + 4z = 2 - 3 + 4 = 3$ **Ans.**]

13. (A)

Sol. $\sin^{-1}x + \tan^{-1}x = k,\ x \in [-1, 1]$ [As, $\sin^{-1}x + \tan^{-1}x$ is an increasing function in $[-1, 1]$]

$\therefore k \in \left[\frac{-3\pi}{4}, \frac{3\pi}{4}\right]$

But $k \in I$; $k = -2, -1, 0, 1, 2.$]

14. (B)

Sol. $\frac{3\pi}{2} < \theta < \frac{5\pi}{2}$ or $\frac{-\pi}{2} + 2\pi < \theta < 2\pi + \frac{\pi}{2}$

or $\frac{-\pi}{2} < \theta - 2\pi < \frac{\pi}{2}$

Now $\tan(\theta - 2\pi) = \tan(-(2\pi - \theta))$

$= -\tan(-(2\pi - \theta)) = \tan\theta,$

hence $\tan^{-1}(\tan\theta) = \tan^{-1}(\tan(\theta - 2\pi))$

As $\tan^{-1}(\tan\theta) = \theta$ is true only if $\theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Hence $\tan^{-1}(\tan(\theta - 2\pi)) = \theta - 2\pi$ as $\frac{-\pi}{2} < \theta - 2\pi < \frac{\pi}{2}$ **Ans.**

15. (C)

Sol. $\tan\left(\tan^{-1}\frac{x}{10} + \tan^{-1}\frac{1}{x+1}\right) = \tan\frac{\pi}{4}$

$\Rightarrow \dfrac{\frac{x}{10} + \frac{1}{x+1}}{1 - \frac{x}{10}\left(\frac{1}{x+1}\right)} = 1 \quad \Rightarrow \frac{x}{10} + \frac{1}{x+1} = 1 - \frac{x}{10}\left(\frac{1}{x+1}\right)$

$\Rightarrow x(x + 1) + 10 = 10(x + 1) - x \Rightarrow x^2 + x + 10 = 10x + 10 - x$

$\Rightarrow x^2 - 8x = 0 \Rightarrow x = 0, 8.$ **Ans.**]

16. (D)

Sol. $S_n = \sum_{r=0}^{n-1} \tan^{-1}\left(\frac{n}{n^2 + r(r+1)}\right)$

$S_n = \sum_{r=0}^{n-1} \tan^{-1}\left(\dfrac{\frac{1}{n}}{1 + \frac{r+1}{n}\cdot\frac{r}{n}}\right)$

$S_n = \sum_{r=0}^{n-1} \tan^{-1}\left(\dfrac{\frac{r+1}{n} - \frac{r}{n}}{1 + \frac{r+1}{n}\cdot\frac{r}{n}}\right)$

$S_n = \sum_{r=0}^{n-1} \tan^{-1}\left(\frac{r+1}{n}\right) - \tan^{-1}\frac{r}{n}$

$S_n = \left(\tan^{-1}\frac{1}{n} - 0\right)$

$+ \left(\tan^{-1}\frac{2}{n} - \tan^{-1}\frac{1}{n}\right)$

$+ \left(\tan^{-1}1 - \tan^{-1}\frac{n-1}{n}\right)$

$\therefore S_n = S_{100} = \frac{\pi}{4}$. **Ans.**]

17. (C)

Sol. The equation $3x^2 + 6x + a = 0$ must have equal roots

So, $D = 0$]

$\Rightarrow 36 - 12a = 0 \Rightarrow a = 3.$ **Ans.**]

18. (B)

Sol. $\tan^{-1}y = \tan^{-1}x + C$

$x = 0;\ y = 1 \Rightarrow C = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}y = \tan^{-1}x + \frac{\pi}{4} \Rightarrow$

note: even $\frac{-\pi}{2} < \tan^{-1}x + \frac{\pi}{4} < \frac{\pi}{2}$;

$\frac{-\pi}{2} < \tan^{-1}x < \frac{\pi}{4}$; $\boxed{-\infty < x < 1} \Rightarrow$ (A)

$x < 1$

Hence $y = \tan\left(\tan^{-1}x + \frac{\pi}{4}\right) = \left(\frac{x+1}{1-x}\right) \Rightarrow x = \frac{y-1}{y+1} < 1 \Rightarrow \frac{2}{y+1} > 0 \Rightarrow y \in (-1, \infty).$]

19. (B)

Sol. $x\cos^{-1}x - 2\cos^{-1}x + 2 - x = 0$

$(x-2)(\cos^{-1}x - 1) = 0$

$x = 2,\ \cos^{-1}x = 1$

(rejected) $\therefore x = \cos 1$

Hence number of solutions is one. **Ans.**

20. (B)

Sol. $1 + \tan^2(\tan^{-1}x) - \left(\sec^2\left(\sec^{-1}x\right) - 1\right)$

$1 + \left(\tan\left(\tan^{-1}x\right)\right)^2 - \left(\sec\left(\sec^{-1}x\right)\right)^2 + 1 =$

$1 + x^2 - x^2 + 1 = 2$. **Ans.**

21. 3

Sol. $\alpha = \frac{\pi}{2},\ \beta = \pi$

$\therefore 2\sin\alpha = 2$ and $\cos\beta = -1$

So, $p = 1,\ q = -2$.

22. 5

Sol. Domain $= [-6, 10) - \{2, 4, 6\}$

$\therefore$ Even integers in the domain are $-6, -4, -2, 0, 8$

23. 2

Sol. Given that, $\tan^{-1}\sqrt{x^2 - x} + \operatorname{cosec}^{-1}\sqrt{1-(x^2-x)} = \frac{\pi}{2}$

For domain, we must have $x^2 - x = 0 \Rightarrow x = 0, 1$.

Also, both $x = 0$ and $x = 1$ satisfies the equation $\sin^{-1}x = 2\tan^{-1}x$.

24. 6

Sol. We must have, $4\sin^2\theta + \sin\theta = -1 + 6\sin\theta$

$\Rightarrow \sin\theta = 1, \frac{1}{4} \Rightarrow$ 6 solutions.

25. 12

Sol. $f(x)$ will be minimum at $x = \frac{\pi}{2} \Rightarrow g(x) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x + \tan^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}x = \cot^{-1}x$

$\Rightarrow \sin^{-1}x = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow x = \frac{1}{\sqrt{1+x^2}} \Rightarrow x^2(1+x^2) = 1 \Rightarrow x^4 + x^2 - 1 = 0$

$\Rightarrow x^2 = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$= 2\left(\frac{\sqrt{5}-1}{4}\right) = 2\sin\left(\frac{\pi}{10}\right) \Rightarrow x$

$= \sqrt{2\sin\left(\frac{\pi}{10}\right)}$

$\therefore \lambda + k = 12$.

26. 38

Sol. Given $\alpha = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$; $x \in [-1, 1]$

$\therefore \alpha = 2\tan^{-1}x\ \forall\ x \in [-1, 1]$

Hence, range of α is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ ....(1)

$\beta = \cos^{-1}\left(\frac{3\cos y - 4\sin y}{10}\right)$; $y \in [0, 2\pi]$

Now, $3\cos y - 4\sin y \in [-5, 5]$

$\therefore$ range of β is $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ ....(2)

Also $\gamma = 2\tan^{-1}(z^2 - 4z + 5),\ z \in R$

$\gamma = 2\tan^{-1}\left((z-2)^2 + 1\right)$

$\therefore$ Range of γ is $\left[\frac{\pi}{2}, \pi\right)$ ....(3)

If $\beta + \gamma$ is minimum, then $\beta = \frac{\pi}{3}$ and $\gamma = \frac{\pi}{2}$

Also, α, β, γ are angles of a triangle.

Now, $\alpha = \frac{\pi}{6},\ \beta = \frac{\pi}{3}$ and $\gamma = \frac{\pi}{2}$

$\alpha = 2\tan^{-1}x = \frac{\pi}{6}$ Þ $x = \tan\frac{\pi}{12} = 2 - \sqrt{3}$

$b = \frac{\pi}{3} \Rightarrow \cos\beta = \frac{3\cos y - 4\sin y}{10} = \frac{1}{2}$

$\therefore \frac{3\cos y}{5} - \frac{4\sin y}{5} = 1 \Rightarrow \cos(y + \theta) = 1$

where $\tan\theta = \frac{4}{3}$

$y + \theta = 2\pi \Rightarrow y = 2\pi - \theta = 2\pi - \tan^{-1}\frac{4}{3}$

$\gamma = \frac{\pi}{2} \Rightarrow \frac{\pi}{2} = 2\tan^{-1}(z^2 - 4z + 5)$

$\therefore z^2 - 4z + 5 = 1 \Rightarrow (z-2)^2 = 0 \Rightarrow z = 2$

Now, x + tan y + z = $\left(2-\sqrt{3}\right)+\left(\frac{-4}{3}\right)$ + 2 = $4 - \frac{4}{3}-\sqrt{3}$ = $\frac{8-3\sqrt{3}}{3}$ = $\frac{8-\sqrt{27}}{3}$ ≡ $\frac{a-\sqrt{b}}{c}$

∴ a = 8, b = 27, c = 3
⇒ a + b + c = 38. **Ans.**

27. 6

Sol. Since range is a subset of $\left(0,\frac{\pi}{2}\right)$. Hence, $x^2 + (k-1)x + 9 > 0 \ \forall\ x \in R$

D < 0
⇒ $(k-1)^2 - 36 < 0$ ⇒ $(k-7)(k+5) < 0$
⇒ $k \in (-5, 7)$
∴ $k_{max} = 6$. **Ans.**

28. 2

Sol. Given, $\frac{\sin^{-1}x^2+\cos^{-1}x}{\cos^{-1}x^2+\sin^{-1}x} = -3$

⇒ $\sin^{-1}x^2 + \cos^{-1}x = -3\cos^{-1}x^2 - 3\sin^{-1}x$
⇒ $\sin^{-1}x^2 + 3\cos^{-1}x^2 + \cos^{-1}x + 3\sin^{-1}x = 0$
⇒ $\pi + 2\cos^{-1}x^2 + 2\sin^{-1}x = 0$

⇒ $\cos^{-1}x^2 + \sin^{-1}x = \frac{-\pi}{2}$

Since, $0 \le \cos^{-1}x^2 \le \frac{\pi}{2}$ and $\frac{-\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

So, $\cos^{-1}x^2 = 0$ ⇒ $x = \pm 1$ and $\sin^{-1}x = \frac{-\pi}{2}$ ⇒ x = – 1.

∴ Only solution is x = α = – 1.
Hence, $(-1)^2 + 2(-1) + 3 = 2$. **Ans.**

29. 2

Sol. $x \in \left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ ⇒ $1 + x = \frac{\pi}{3}$ ⇒ $x = \frac{\pi}{3}-1$

$x \in \left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ ⇒ $1+\pi-x = \frac{\pi}{3}$ ⇒ $x = \frac{2\pi}{3}+1$

∴ Answer = $\frac{\pi}{3}-1$, $\frac{2\pi}{3}+1$.

30. 5

Sol. $\sum_{k=1}^{n}\cot^{-1}(1+k+k^2) = \tan^{-1}(n+1) - \tan^{-1}1$

take $\lim_{n\to\infty}$, we get R.H.S. = $\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

= $\tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{1}{\beta}\right)$

⇒ $(\alpha-1)(\beta-1) = 2$
∴ α = 3, β = 2 (or α = 2, β = 3)
∴ Answer = 5.