JEE MAIN : CHAPTER WISE TEST PAPER-3

SUBJECT :- MATHEMATICS **DATE**.....

CLASS :- 12th **NAME**.....

CHAPTER :- INVERSE TRIGONOMETRIC FUNCTIONS **SECTION**.....

(SECTION-A)

1. If α and β ($\alpha > \beta$) are roots of the equation $x^2 - \sqrt{2}x + \sqrt{3-2\sqrt{2}} = 0$ then the value of ($\cos^{-1}\alpha + \tan^{-1}\alpha + \tan^{-1}\beta$) is equal to

(A) $\frac{3\pi}{8}$ (B) $\frac{5\pi}{8}$ (C) $\frac{7\pi}{8}$ (D) $\frac{\pi}{3}$

2. Let f(x) = $\cos(\tan^{-1}x)$, then range of f(x) is equal to

(A) $\left\{x \mid \frac{-\pi}{2} < x < \frac{\pi}{2}\right\}$ (B) $\{x \mid 0 < x \le 1\}$

(C) $\{x \mid 0 \le x \le 1\}$ (D) $\{x \mid -1 \le x \le 1\}$

3. The value of $\sum_{r=2}^{\infty} \tan^{-1}\left(\frac{1}{r^2 - 5r + 7}\right)$, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{4}$

4. Let f(x) = $\sin^{-1}(\sin x)$, g(x) = $\cos^{-1}(\cos x)$, then which one of the following **incorrect**?

(A) $f(x) = g(x)\ \forall\ x \in \left(0, \frac{\pi}{4}\right)$

(B) $f(x) < g(x)\ \forall\ x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

(C) $f(x) < g(x)\ \forall\ x \in \left(\pi, \frac{5\pi}{4}\right)$

(D) $f(x) > g(x)\ \forall\ x \in \left(\frac{3\pi}{2}, 2\pi\right)$

5. The set of lines $x\tan^{-1}c + y\cot^{-1}c + 2 = 0$ where $c \in (0, 1)$ are concurrent at

(A) $\left(\frac{1}{\pi}, \frac{1}{\pi}\right)$ (B) $\left(\frac{4}{\pi}, \frac{4}{\pi}\right)$

(C) $\left(\frac{-1}{\pi}, \frac{-1}{\pi}\right)$ (D) $\left(\frac{-4}{\pi}, \frac{-4}{\pi}\right)$

6. If $\cot\left(\sin^{-1}\sqrt{\frac{13}{17}}\right) = \sin(\tan^{-1}x)$, then x is equal to

(A) $\frac{2}{\sqrt{13}}$ (B) $\frac{-2}{3}$ (C) $\frac{-2}{\sqrt{13}}$ (D) $\frac{2}{3}$

7. Let k = $\tan^{-1}\left(\frac{\sin\frac{\pi}{18} + \sin\frac{2\pi}{18}}{\cos\frac{\pi}{18} + \cos\frac{2\pi}{18}}\right)$, then k equals

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{12}$ (C) $\frac{\pi}{15}$ (D) $\frac{\pi}{4}$

8. Number of solutions of the equation $|\sin^{-1}(\sin x)| = \cos x$, for $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is equal to

(A) 0 (B) 2 (C) 4 (D) 6

9. The value of $\sum_{r=1}^{\infty} \cot^{-1}\left(\frac{r^2}{2} + \frac{15}{8}\right)$ is equal to

(A) $\tan^{-1}1$ (B) $\tan^{-1}2$

(C) $\tan^{-1}3$ (D) $\tan^{-1}4$

10. Let f(x) = $\begin{cases} \sqrt{10 - x^2}; & -3 < x < 3 \\ 2 - e^{x-3}; & 3 \le x \le 3 + \ln 3 \end{cases}$.

Which one of the following statement is **incorrect**?

(A) f(x) Not one - one function.

(B) Number of solutions of the equation f(x) = 1 is two.

(C) Range of f(x) is $\left[-1, \sqrt{10}\right]$.

(D) f(x) is not injective.

11. Number of integral ordered pair(s) (x, y) satisfying the equation $\tan^{-1}(x + 2011) + \tan^{-1}\left(\frac{1}{y + 2012}\right) = \tan^{-1}2$, is equal to

(A) 1 (B) 2 (C) 3 (D) 4

12. If $(\sin^{-1}x)^3 + (\sin^{-1}y)^3 + (\sin^{-1}z)^3 = \frac{(3\pi^3)}{8}$, then the value of (2x – 3y + 4z) is equal to

(A) 2 (B) 3 (C) 4 (D) 9

13. The number of integral values of k for which the equation $\sin^{-1}x + \tan^{-1}x = k$ has a solution, is

(A) 5 (B) 7 (C) 11 (D) 13

14. The value of $\tan^{-1}(\tan\theta)$ where $\frac{3\pi}{2} < \theta < \frac{5\pi}{2}$ is

(A) θ (B) $\theta - 2\pi$

(C) $\frac{-3\pi}{2}$ (D) $\theta - \frac{5\pi}{2}$

15. Number of positive integers x satisfying the equation $\tan\left(\tan^{-1}\frac{x}{10} + \tan^{-1}\frac{1}{x+1}\right) = \tan\frac{\pi}{4}$, is

(A) 0 (B) 1 (C) 2 (D) 3

16. Let $S_n = \sum_{r=0}^{n-1} \cos^{-1}\left(\frac{n^2+r^2+r}{\sqrt{n^4+r^4+2r^3+2n^2r^2+2n^2r+n^2+r^2}}\right)$. Then the value of S_{100}, is

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

17. Let $f: R \to \left[0, \frac{\pi}{2}\right)$ be defined by $f(x) = \tan^{-1}(3x^2 + 6x + a)$. If f (x) is an onto function then the value of a is

(A) 1 (B) 2 (C) 3 (D) 4

18. Let y = f (x) be a function satisfing the equation $\tan^{-1}y = \tan^{-1}x + C$ where y = 1 when x = 0. Range of the function f (x), is

(A) R – {–1} (B) (–1, ∞)
(C) [1, ∞) (D) (–∞, 1]

19. Number of real solutions satisfying the equation $x\cos^{-1}x + 2 = x + 2\cos^{-1}x$, is

(A) 0 (B) 1 (C) 2 (D) 3

20. For any real number $x \geq 1$ the expression $\sec^2(\tan^{-1}x) - \tan^2(\sec^{-1}x)$ is equal to

(A) 1 (B) 2 (C) $2x^2$ (D) $2\sqrt{2}$

(SECTION-B)

21. Let $\alpha = 2\tan^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5}$ and $\beta = \sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \cot^{-1}\frac{16}{63}$ be such that $2\sin\alpha$ and $\cos\beta$ are roots of the equation $x^2 - px + q = 0$, then find $(p - q)$.

22. Two functions f(x) and g(x) are defined as $f(x) = \log_3\left|\frac{x-2}{x^2-10x+24}\right|$ and $g(x) = \sin^{-1}\left(\frac{2[x]-3}{15}\right)$ then find the number of even integers for which $(f(x)+g(x))$ is defined.

[**Note :** [k] denotes greatest integer less than or equal to k.]

23. Find the number of values of x satisfying simultaneously $\sin^{-1}x = 2\tan^{-1}x$ and $\tan^{-1}\sqrt{x(x-1)} + \text{cosec}^{-1}\sqrt{1+x-x^2} = \frac{\pi}{2}$.

24. Find the number of solutions of equation $\sin^{-1}(4\sin^2\theta + \sin\theta) + \cos^{-1}(-1 + 6\sin\theta) = \frac{\pi}{2}$, in $\theta \in [0, 5\pi]$.

25. Consider, $f(x) = |x - 1| + |2x - \pi| + |x - 3|$ and $g(x) = \sin^{-1}x + \tan^{-1}x$. The value of x for which $f(g(x))$ is minimum is $\sqrt{k\sin\frac{\pi}{\lambda}}$ where k and $\lambda \in N$ then find the value of $(k + \lambda)$.

26. Consider, $\alpha = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $x \in [-1, 1]$, $\beta = \cos^{-1}\left(\frac{3\cos y - 4\sin y}{10}\right)$, $y \in [0, 2\pi]$ and $\gamma = 2\tan^{-1}(z^2 - 4z + 5)$, $z \in R$. If α, β and γ are interior angles of a triangle such that $(\beta + \gamma)$ is minimum then $x + \tan y + z = \frac{a-\sqrt{b}}{c}$ where, a, b, c ∈ N. Find the least value of (a + b + c).

27. Let f be a function defined by $f(x) = \tan^{-1}(x^2 + kx + 9 - x)$. If range of f(x) lies in the interval $\left(0, \frac{\pi}{2}\right)$ for all values of $x \in R$, then find the maximum integral value of k.

28. If $x = \alpha$ satisfies the equation $\frac{\sin^{-1}x^2 + \cos^{-1}x}{\cos^{-1}x^2 + \sin^{-1}x} = -3$, then find the value of $(\alpha^2 + 2\alpha + 3)$.

29. Find the number of points $x \in \left[\frac{-\pi}{2}, \frac{3\pi}{2}\right]$ satisfying the equation $1 + \sin^{-1}(\sin x) = \frac{\pi}{3}$.

30. For any real number x, let $\cot^{-1}x$ denote the unique real number θ in $(0, \pi)$ such that $\cot\theta = x$.

If $\lim_{n\to\infty}\sum_{k=1}^{n}\cot^{-1}(1+k+k^2) = \cot^{-1}(\alpha) + \cot^{-1}(\beta)$, where α, β are prime numbers, then find $(\alpha + \beta)$.