JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 12th
CHAPTER :- FUNCTION
PAPER CODE :- CWT-2

ANSWER KEY

1.	(D)	**2.**	(D)	**3.**	(C)	**4.**	(B)	**5.**	(D)	**6.**	(C)	**7.**	(B)
8.	(B)	**9.**	(A)	**10.**	(A)	**11.**	(D)	**12.**	(D)	**13.**	(B)	**14.**	(B)
15.	(D)	**16.**	(A)	**17.**	(D)	**18.**	(C)	**19.**	(C)	**20.**	(A)	**21.**	8
22.	5	**23.**	2	**24.**	0	**25.**	5	**26.**	4	**27.**	3	**28.**	2
29.	2	**30.**	64										

SOLUTIONS

1. (D)

Sol. Clearly (f + g) (x) = f(x) + g(x) = 2 $\forall$ x $\in$ R = constant function
Hence the function (f + g) (x) defined on R $\to$ R is neither surjective nor injective.]

2. (D)

Sol. {(20, 1), (19, 3), (18, 5), (17, 7), (16, 9), (15, 11), (14, 13), (13, 15), (12, 7), (11, 19), (10, 21), (9, 23), (8, 25), (7, 27), (6, 29), (5, 31), (4, 33), (3, 35), (2, 37), (1, 39)}
Clearly R is neither reflexive, nor symmetric nor transitive.

3. (C)

Sol. B We have $[2x^2] + x - n = 0$

$\Rightarrow$ x has to be an integer.

$\Rightarrow$ $n = 2x^2 + x = x(2x + 1)$

$\therefore$ n can be 21, 36, 55, 78 corresponding to x = 3, 4, 5, 6.

Hence, sum of all possible values of n is equal to 190. Ans.

Note: If x is negative also then answer is 435.]

4. (B)

Sol. We have $y = \dfrac{x-1}{p - x^2 + 1} \Rightarrow py - x^2y + y = x - 1 \Rightarrow x^2y + x - y(p + 1) - 1 = 0$

As, x $\in$ R, so $D \ge 0 \Rightarrow 1 + 4y\left(y(p+1)+1\right) \ge 0 \Rightarrow 4y^2(p + 1) + 4y + 1 \ge 0$

Since $y \ne \left[-1, \dfrac{-1}{3}\right]$

So, $4y^2(p + 1) + 4y + 1 < 0 \quad \forall\, y \in \left[-1, \dfrac{-1}{3}\right] \Rightarrow (2y + 1)^2 + 4y^2 p < 0$

$\Rightarrow p < -1\left(\dfrac{2y+1}{2y}\right)^2 \quad \forall\, y \in \left[-1, \dfrac{-1}{3}\right]$

Hence $p < -\left(\dfrac{2y+1}{2y}\right)^2\Bigg|_{\min}$

Now max. value of $\left(\dfrac{2y+1}{2y}\right)^2$ occurs at y = –1 and is equal to $\dfrac{-1}{4}$.

$\therefore \quad p < \dfrac{-1}{4} \Rightarrow$ B Ans.]

5. (D)

Sol. We have,

$$f(x) = 4\cos^4\left(\frac{x-\pi}{4\pi^2}\right) - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$$

$$= \left(2\cos^2\left(\frac{x-\pi}{4\pi^2}\right)\right)^2 - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$$

$$= \left(1+\cos\left(\frac{x-\pi}{2\pi^2}\right)\right)^2 - 2\cos\left(\frac{x-\pi}{2\pi^2}\right) = 1 + \cos^2\left(\frac{x-\pi}{2\pi^2}\right)$$

Clearly, period of f(x) = $\dfrac{\pi}{\dfrac{1}{2\pi^2}} = 2\pi^3$. **Ans.**]

6. (C)

Sol. $g(x^3 + 1) = x^6 + x^3 + 2 = (x^3 + 1)^2 - x^3 + 1 = (x^3 + 1)^2 - (x^3 + 1 - 1) + 1 = (x^3 + 1)^2 - (x^3 + 1) + 2$

Put $x^3 + 1 = t$

So, $g(t) = t^2 - t + 2$

$\Rightarrow$ $g(x^2 - 1) = (x^2 - 1)^2 - (x^2 - 1) + 2 = x^4 - 3x^2 + 4$. **Ans.**]

7. (B)

Sol. $f : [1, \infty) \to [1, \infty)$

$y = f(x) = 2^{x(x-1)} \quad \Rightarrow \quad \log_2 4 = x(x-1)$

$\log_2 y = x(x-1)$

$x^2 - x - \log_2 y = 0$

$x = \dfrac{1 \pm \sqrt{1+4\log_2 y}}{2}$

$f^{-1}(x) = \dfrac{1+\sqrt{1+4\log_2 x}}{2}$]

8. (B)

Sol. $g(x) = 1 + \{x\}$

$f(x) = \text{sgn}(x)$

$(f\,og)(x) = f(1+\{x\}) = \text{sgn}(1+\{x\}) = 1$]

9. (A)

Sol. $f : [1, \infty) \longrightarrow [2, \infty)$

$f(x) = x + \dfrac{1}{x} \quad \Rightarrow y = \dfrac{x^2+1}{x}$

$x^2 - xy + 1 = 0$

$x = \dfrac{y \pm \sqrt{y^2-4}}{2}$

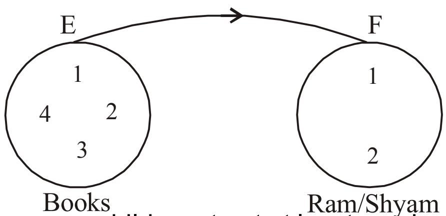
$x = \dfrac{y - \sqrt{y^2-4}}{2}$ (rejected) as $x \to \infty$, $y \to \infty$

$f^{-1}(x) = \dfrac{x + \sqrt{x^2-4}}{2}$.]

10. (A)

Sol. Total $2^4 = 2$ = 14 Ans.

(4 books to be distributed in 2 children so that every



child must get at least one book)

T – Into]

11. (D)

Sol. $y = f(x) = \dfrac{\alpha x}{x+1}$; $x \neq -1$

$f[f(x)] = x$

$f\left[\dfrac{\alpha x}{x+1}\right] = \dfrac{\alpha \cdot \alpha x}{(x+1)\left(\dfrac{\alpha x}{n+1}+1\right)} = x$

$\Rightarrow \quad \dfrac{\alpha^2 x}{\alpha x + x + 1} = \dfrac{x}{1}$

$\dfrac{\alpha^2 x}{x(\alpha+1)+1} = \dfrac{x}{1}$

comparing $\alpha^2 = 1$ & $\alpha + 1 = 0$

$\therefore \quad \alpha = -1$]

12. (D)

Sol. $f(x) = (x - 5a)^2 + 5 - a. \Rightarrow 5 - a = 1 \Rightarrow a = 4.$]

13. (B)

Sol. Let $y = g(x) \Rightarrow x = g^{-1}(y)$

$y = a\, f\left(\dfrac{x}{a}+1\right)$

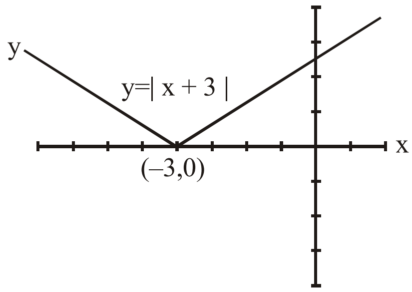
$\therefore \; f\left(\dfrac{x}{a}+1\right) = \dfrac{y}{a} \Rightarrow \dfrac{x}{a} + 1 = f^{-1}\left(\dfrac{y}{a}\right)$

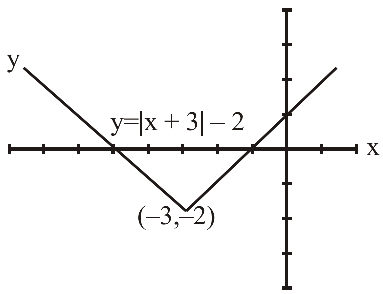
$\Rightarrow x = a\left[f^{-1}\left(\dfrac{y}{a}\right) - 1\right]$

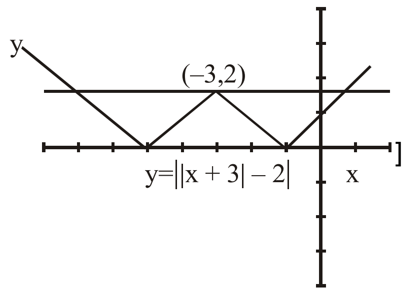
$\Rightarrow g^{-1}(x) = a\left(f^{-1}\left(\dfrac{x}{a}\right) - 1\right)$. **Ans.**]

14. (B)

Sol. The following sequence of graphs gives a progression that leads directly to the answer.







15. (D)

Sol. hogof(x) = $\cos^{-1}\left(\sqrt{\sin^2 x}\right)$

$= \cos^{-1}(\sin x) = \frac{\pi}{2} - x$

gofoh(x) = $\sqrt{\sin^2(\cos^{-1}x)} = \sqrt{1-x^2}$

fohog(x) = $\sin^2(\cos^{-1}\sqrt{x}) = 1 - x$.

Thus no two composites are equal.

16. (A)

Sol. $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$

$= \frac{(x^2+1)(x^2-2x+2)}{2(x^2-x)+1}$

$= \frac{(x^2+1)[(x-1)^2+1]}{2\left(x-\frac{1}{2}\right)^2+\frac{1}{2}} > 0$

for all x in $(-\infty, \infty)$ and log x is real for all x > 0. It follows that fog(x) = log g(x) is defined for all real x.

17. (D)

Sol.

$$f\left(g\left(h(1)\right)\right) + g\left(h\left(f(-3)\right)\right) + h\left(f\left(g(-1)\right)\right)$$

= 2 + 5 + 0 = 7]

18. (C)

Sol. $f(x) = \frac{1}{g(x)}$; $f'(x) = \frac{-1}{g^2(x)} \cdot g'(x)$

$f'(1) = \frac{-1}{g^2(1)}\, g'(1) = \frac{-1}{9}\left(\frac{1}{f'(3)}\right) = \frac{-1}{9}\left(\frac{1}{2}\right)$

$= \frac{-1}{18}$. **Ans.**]

19. (C)

Sol. $yx + 2y = x^2 + x + 7$

$x^2 + x(1-y) + 7 - 2y = 0$ as $x \in R$

$D \ge 0$

$(1-y)^2 - 4(7-2y) \ge 0$

$y^2 + 6y - 27 \ge 0$

$(y+9)(y-3) \ge 0$

$y \in (-\infty, -9] \cup [3. \infty)$

$\therefore$ Smallest positive value of f(x) is 3.]

20. (A)

Sol. $f(x) = \frac{(x-1)(x-2)}{(x-3)(x-1)} = \frac{x-2}{x-3}$, $x \ne 1, 3$

Domain of function = R – {1, 3}.

Range of function = $R - \left\{\frac{1}{2}, 1\right\}$ **Ans.**]

21. 8

Sol. Range of f(x) is [0, 7)

Hence, d = 7

Now, one root of P(x) is less than 1 and other root greater than 2.

Hence, P(1) < 0 $\Rightarrow$ 21 – 3m < 0 $\Rightarrow$ m > 7

and P(2) < 0 $\Rightarrow$ 24 – 2m < 0 $\Rightarrow$ m > 12

Hence, m > 12.

$\therefore$ Least integral value of m is 13

$\Rightarrow$ (k – 5) = 8. **Ans.**]

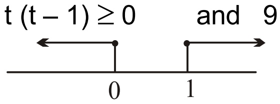
22. 5

Sol. $f(x) = \sqrt{x^2 - |x|} + \frac{1}{\sqrt{9-x^2}}$

Let $|x| = t$

$f(x) = \sqrt{t^2 - t} + \frac{1}{\sqrt{9-t^2}}$

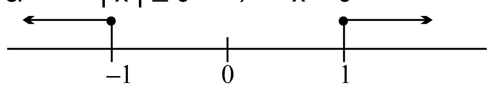
$t(t-1) \ge 0$ and $9 - t^2 > 0$

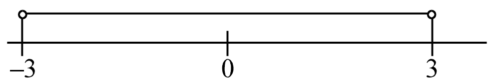


and $t \in (-3, 3)$

$|x| \ge 1$ $x \ge 1$ or $x \le -1$

or $|x| \le 0 \Rightarrow x = 0$





$(-3, -1] \cup [1, 3) \cup \{0\}$

Hence number of integer in the domain of function is 5 **Ans.**]

23. 2

Sol. y

$= \dfrac{4}{[\tan x]^2 + 5[\tan x] + 6}$

$= \dfrac{4}{([\tan x]+2)([\tan x]+3)}$

$\because x \in \left(0, \dfrac{\pi}{2}\right)$

$\Rightarrow$ tan x $\in (0, \infty)$

$\Rightarrow$ [tan x] = 0, 1, 2, 3, ……

Let [tan x] = n, n $\in$ N $\cup \{0\}$

$= \dfrac{4}{(n+2)(n+3)}$

$y = 4\left(\dfrac{1}{n+2} - \dfrac{1}{n+3}\right)$

Required sum of all values of y =

$4\left(\dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots\ldots\infty\right)$ = 2 **Ans.**]

24. 0

Sol. For g(x) to be bijective

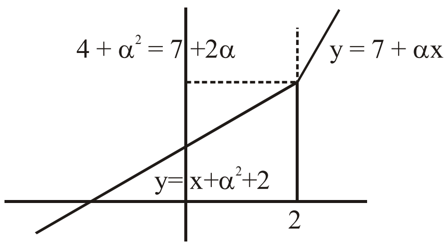
Clearly $\alpha > 0$

and $4 + \alpha^2 = 7 + 2\alpha$

$\Rightarrow$ $\alpha^2 - 2\alpha - 3 = 0$

$\Rightarrow$ $(\alpha - 3)(\alpha + 1) = 0$

$\therefore$ $g(x) = \begin{cases} x + 11, & x \le 2 \\ 3x + 7, & x > 2 \end{cases}$



$\because$ f (x) is inverse of g(x)

and $f(11) = g^{-1}(11)$

and $g(x) = 11 \Rightarrow x = 0 \Rightarrow g^{-1}(11) = 0$]

25. 5

Sol. $\because$ $|x + y| \le |x| + |y|$

But $|f(x) + (x^2 + 1)| \ge |f(x)| + |x^2 + 1|$

$\therefore$ $|f(x) + (x^2 + 1)| = |f(x)| + |x^2 + 1|$ will be true iff $f(x) \cdot (x^2 + 1) \ge 0$

$\Rightarrow f(x) \ge 0$

But $f(x) \le 0$, $\therefore$ f(x) = 0

$\therefore \sum_{r=1}^{5}(1 + f(r)) = \sum_{r=1}^{5} 1 = 5$ **Ans.**]

26. 4

Sol. Let $\sqrt{x-1} + \sqrt{17-x} = t$

$\Rightarrow x - 1 + 17 - x + 2\sqrt{64 - (x-9)^2} = t^2$

$\Rightarrow 16 + 2\sqrt{64 - (x-9)^2} = t^2$

$\Rightarrow f(t) = \sqrt{4 + t^2}$, $t \in [4, 4\sqrt{2}]$

$f(t) = 5 \Rightarrow 4 + t^2 = 25 \Rightarrow t = \sqrt{21}$

$\Rightarrow f^{-1}(5) = \sqrt{21} \Rightarrow [f^{-1}(5)] = 4$]

27. 3

Sol. f(x) = mx + 4

g(x) = m'(x – 5) + 17

$g(f(x))$ = m'[(mx + 4) – 5] + 17 = x

$\because$ Identify function

$\Rightarrow m = \dfrac{1}{17}$, m' = 17

$\Rightarrow f(x) = \dfrac{1}{17}x + 4$

f(136) = 12 = 4k $\Rightarrow$ k = 3]

28. 2

Sol. $\dfrac{-b}{2a} \le 1$ and f (1) = 2

$k^2 - 3k + 1 \ge -1 \Rightarrow k^2 - 3k + 2 \ge 0 \Rightarrow (k-1)(k-2) \ge 0$

$\Rightarrow k \in (-\infty, 1] \cup [2, \infty)$

Also f (1) = 2

$1 + 2(k^2 - 3k + 1) + k^2 - 1 = 2 \Rightarrow 3k^2 - 6k = 0$

$\Rightarrow$ k = 0 or 2

Hence, number of integral values of k are 2 **Ans.**]

29. 2

Sol. $\because f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}} = \begin{cases} \frac{e^x - e^{-x}}{2e^x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

$\therefore$ for $x \geq 0$

$y = f(x) = \frac{e^x - e^{-x}}{2e^x}$

$y = \frac{1}{2}(1 - e^{-2x})$

$\because \quad 0 \leq x < \infty$

$\Rightarrow 0 < e^{-2x} \leq 1$

$\Rightarrow -1 \leq -e^{-2x} < 0$

$\Rightarrow 0 \leq 1 - e^{2x} < 1$

$\Rightarrow 0 \leq \frac{1-e^{2x}}{2} < \frac{1}{2}$

$\therefore R_f = \left[0, \frac{1}{2}\right)$

$\therefore$ 5a + 4b = 2 **Ans.**]

30. 64

Sol. Given, f(x+f(x)) = 4f(x) & f(1) = 4
Substituting X = 1 we get
(1+f(1))=4f(1) $\Rightarrow$ f(1+4)=16 $\Rightarrow$ f(5)=16
Now substitute x = 5, f(5 + f(5)) = 4f(5) $\Rightarrow$ f(5+16) = 64 = f(21)