JEE MAIN : CHAPTER WISE TEST PAPER-2

SUBJECT :- MATHEMATICS **DATE...**
CLASS :- 12^{th} **NAME...**
CHAPTER :- FUNCTION **SECTION...**

(SECTION-A)

1. Let two functions f(x) and g(x) are defined on $R \to R$ such that f (x)

$= \begin{cases} x^2, & x \in \text{irrational} \\ 2-x^2, & x \in \text{rational} \end{cases}$

and g(x) = $\begin{cases} 2-x^2, & x \in \text{irrational} \\ x^2, & x \in \text{rational} \end{cases}$.

Then the function $f + g : R \to R$ is
(A) injective as well as surjective.
(B) injective but not surjective.
(C) surjective but not injective.
(D) neither surjective nor injective.

2. Let R be the relation defined on the set of natural numbers N as
R = { (x, y) | x ∈ N, y ∈ N, xRy ⇒ 2x + y = 41}, then which one of the following holds good?
(A) R is reflexive
(B) R is symmetric
(C) R is transitive
(D) R is neither reflexive nor symmetric nor transitive

3. The sum of all possible values of n where $n \in N$, $x > 0$ and $10 < n \le 100$ such that the equation $[2x^2] + x - n = 0$ has a solution, is equal to
[**Note:** [x] denotes largest integer less than or equal to x.]
(A) 150 (B) 175 (C) 190 (D) 210

4. If the range of the function $f(x) = \frac{x-1}{p-x^2+1}$ does not contain any values belonging to the interval $\left[-1, \frac{-1}{3}\right]$ then the true set of values of p, is
(A) $(-\infty, -1)$ (B) $\left(-\infty, \frac{-1}{4}\right)$
(C) $(0, \infty)$ (D) $(-\infty, 0)$

5. The fundamental period of the function $f(x) = 4\cos^4\left(\frac{x-\pi}{4\pi^2}\right) - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$ is equal to
(A) π^3 (B) $4\pi^2$ (C) $3\pi^2$ (D) $2\pi^3$

6. If $g(x^3 + 1) = x^6 + x^3 + 2$, then the value of $g(x^2 - 1)$ is
(A) $x^4 - 3x^2 + 3$ (B) $x^4 + x^2 + 4$
(C) $x^4 - 3x^2 + 4$ (D) $x^4 + x^2 + 2$

7. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is
(A) $\left(\frac{1}{2}\right)^{x(x-1)}$
(B) $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 x}\right)$
(C) $\frac{1}{2}\left(1 - \sqrt{1 + 4\log_2 x}\right)$
(D) not defined

8. Let $g(x) = 1 + x - [x]$ & $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$.Then for all x, f (g (x)) is equal to
(A) x (B) 1 (C) f (x) (D) g (x)

9. If $f : [1, \infty) \to [2, \infty)$ is given by, $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals :
(A) $\frac{x + \sqrt{x^2 - 4}}{2}$ (B) $\frac{x}{1 + x^2}$
(C) $\frac{x - \sqrt{x^2 - 4}}{2}$ (D) $1 - \sqrt{x^2 - 4}$

10. Let E = {1, 2, 3, 4} & F = {1, 2}. Then the number of onto functions from E to F is
(A) 14 (B) 16 (C) 12 (D) 8

11. Let $f(x) = \frac{\alpha x}{x+1}$, $x \ne -1$. Then for what value of α is f (f (x)) = x ?
(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 1 (D) –1.

12. Let $f : R \to [1, \infty)$ be a function defined by $f(x) = x^2 - 10ax + 5 - a + 25a^2$. If f(x) is surjective on R, then the value of a is
(A) 0 (B) 1 (C) 2 (D) 4

13. Let f be a bijective function and $a \ne 0$, then the function $g(x) = a\ f\left(\frac{x+a}{a}\right)$ has an inverse function which is
(A) $\frac{1}{a} f^{-1}(x - 1)$ (B) $a\left(f^{-1}\left(\frac{x}{a}\right) - 1\right)$
(C) $a f^{-1}\left(\frac{x}{a}\right) - 1$ (D) $\frac{1}{a} f^{-1}(ax - 1)$

14. If the equation $||x+3|-2| = p$, where p is a constant integer has exactly three distinct solutions, then the number of integral values of p, is
(A) 0 (B) 1 (C) 2 (C) 4

15. If $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ and $h(x) = \cos^{-1}x$, $0 \le x \le 1$, then -
(A) hogof(x) = gofoh(x)
(B) gofoh(x) = fohog(x)
(C) fohog(x) = hogof(x)
(D) None of these

16. Let f(x) = log x and $g(x) = \frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$. The domain of the composite function fog(x) is -
(A) $(-\infty,\infty)$ (B) $[0, \infty)$
(C) $(0, \infty)$ (D) $[1, \infty)$

17. Consider f, g, h be real-valued functions defined on R. Let $f(x) - f(-x) = 0$ for all $x \in R$, $g(x) + g(-x) = 0$ for all $x \in R$ and $h(x) + h(-x) = 0$ for all $x \in R$.
If $f(1) = 0$, $f(4) = 2$, $f(3) = 6$, $g(1) = -1$, $g(-2) = 4$, $g(3) = 5$, and $h(1) = 2$, $h(3) = 5$, $h(6) = 3$.
Then the value of $f(g(h(1)))+g(h(f(-3)))+h(f(g(-1)))$ is equal to
(A) – 1 (B) 1 (C) – 7 (D) 7

18. Let f(x) be a one-to-one function such that f(1) = 3, f(3) = 1, f '(1) = – 4 and f '(3) = 2. If $g = f^{-1}$, then the slope of the tangent line to $\frac{1}{g}$ at x = 1 is
(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{-1}{9}$ (C) $\frac{-1}{18}$ (D) $\frac{1}{32}$

19. The smallest positive integral value of $f(x) = \frac{x^2+x+7}{x+2}$, $x \in R$ is equal to
(A) 1 (B) 2 (C) 3 (D) 4

20. The sum of all real numbers which are not in the range of $f(x) = \frac{x^2-3x+2}{x^2-4x+3}$ is equal to
(A) $\frac{3}{2}$ (B) $\frac{1}{2}$
(C) 1 (D) $\frac{5}{2}$

(SECTION-B)

21. Let d be the number of integers in the range of the function $f(x) = \begin{cases} 4, & \text{if } -4 \le x < -2 \\ |x|, & \text{if } -2 \le x < 7 \\ \sqrt{x}, & \text{if } 7 \le x < 14 \end{cases}$.
Also roots of $P(x) = x^2 + mx - 4m + 20$ are α and β.
If $\alpha < \frac{d-3}{4} < \frac{d-3}{2} < \beta$ and the smallest integral value of m is k, then find the value of $(k-5)$.

22. Find the number of integers in the domain of the function $f(x) = \sqrt{x^2-|x|} + \frac{1}{\sqrt{9-x^2}}$.

23. The sum of all different values of y satisfying the equation $y([\tan x]^2 + 5[\tan x] + 6) = 4$, where $x \in \left(0, \frac{\pi}{2}\right)$ and [k] denotes greatest integer value less than or equal to k,is

24. Let a bijective function $g: R \to R$ be defined as $g(x) = \begin{cases} x+\alpha^2+2, & x \le 2 \\ 7+\alpha x, & x > 2 \end{cases}$.
If graph of y = f(x) is reflection of graph of y = g(x) w.r.t. line y = x, then find f(11).

25. Let f(x) be a real valued function such that $|f(x) + x^2 + 1| \ge |f(x)| + |x^2 + 1|$ and $f(x) \le 0$, then find the absolute value of $\sum_{r=1}^{5}(1+f(r))$.

26. If f: [4, a] → A is a bijective function and defined by $f\left(\sqrt{x-1}+\sqrt{17-x}\right) = \sqrt{20+2\sqrt{64-(x-9)^2}}$, then find the value of $\left[f^{-1}(5)\right]$.
[**Note :** [k] denotes greatest integer less than or equal to k.]

27. f(x) and g(x) are linear functions such that for all x, f(g(x)) and g(f(x)) are identity functions, if f(0) = 4, g(5) = 17 and f(136) = 4k. Then find the value of k.

28. Let $f: [1, \infty) \to [2, \infty)$ defined by $f(x) = x^2 + 2(k^2 - 3k + 1)x + k^2 - 1$. If f(x) both injective and surjective then find the number of all possible integral value(s) of k.

29. Let f be a real valued function defined by $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$, range of f is [a, b), then find the value of (5a + 4b).

30. Suppose f is a real valued function satisfying f(x + f(x)) = 4 f(x) and f(1) = 4. The value of f(21) is -