JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS

CLASS :- 12^{th} **PAPER CODE :- CWT-1**

CHAPTER :- RELATION

ANSWER KEY

1.	(C)	**2.**	(D)	**3.**	(A)	**4.**	(C)	**5.**	(C)	**6.**	(C)	**7.**	(B)
8.	(B)	**9.**	(B)	**10.**	(B)	**11.**	(C)	**12.**	(B)	**13.**	(D)	**14.**	(C)
15.	(C)	**16.**	(D)	**17.**	(A)	**18.**	(A)	**19.**	(B)	**20.**	(A)	**21.**	5
22.	4	**23.**	512	**24.**	0	**25.**	4	**26.**	1	**27.**	7	**28.**	1
29.	4	**30.**	1										

SOLUTIONS

1. (C)

Sol. B U C = {c, d, e} , $B \cap C$ = {d}

$A \cup (B \cup C) = \phi$

$A \cup (B \cap C)$ = {a, b, d}

A × $(B \cup C)$= {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)}

A × $(B \cap C)$ = {(a, d), (b, d)}

2. (D)

Sol. $R_1 \rightarrow$ Domain = {1, 3, 5}
Range = {3, 5, 7}
so R_1 is a relation

$R_2 \rightarrow$ Domain = {1,2, 3, 4, 5}
Range = {1, 3, 5}
so R_2 is a relation

$R_3 \rightarrow$ Domain = {1, 3, 5}
Range = {1, 3, 5, 7}
so R_3 is a relation

$R_4 \rightarrow$ Domain = {1, 2, 7} $\not\subset$ X
so R_4 is not a relation

3. (A)

Sol. $xR_2y \Leftrightarrow x \geq y$ is not symmetric relation

$xR_3y \Leftrightarrow x / y$ is not symmetric relation

$xR_4y \Leftrightarrow x < y$ is not symmetric relation

$xR_1y \Leftrightarrow x^2 = y^2$ is reflexive, symmetric and transitive so equivalence relation

$xR_4y \Leftrightarrow x < y$ is not symmetric relation

$xR_1y \Leftrightarrow x^2 = y^2$ is reflexive, symmetric and transitive so equivalence relation

4. (C)

Sol. Since $x \nless x$, therefore *R* is not reflexive. Also $x < y$ does not imply that $y < x$, So *R* is not symmetric. Let xRy and yRz. Then, $x < y$ and $y < z \Rightarrow x < z$ *i.e.*, xRz. Hence *R* is transitive.

5. (C)

Sol. *x* is a brother of *y*, *y* is also brother of *x*.
So, it is symmetric. Clearly it is transitive.

6. (C)

Sol. Since (1, 1)$\notin R$ so, is not reflexive.
Now (1, 2) $\in R$ but (2,1) $\notin R$, therefore *R* is not symmetric Clearly *R* is transitive.

7. (B)

Sol. The void relation *R* on *A* is not reflexive as (*a*, *a*)$\notin R$ for any $a \in A$. The void relation is symmetric and transitive.

8. (B)

Sol. For any $a \in R$, we have $a \geq a$, Therefore the relation *R* is reflexive but it is not symmetric as (2, 1) $\in R$ but (1, 2) $\notin R$. The relation *R* is transitive also, because $(a, b) \in R, (b, c) \in R$ imply that $a \geq b$ and $b \geq c$ which is turn imply that $a \geq c$.

9. (B)

Sol. For any a $\in$ R, we have a $\geq$ a, Therefore the relation R_1 is reflexive but it is not symmetric as (2, 1) $\in R_1$ but (1, 2) $\notin R_1$. The relation R_1 is transitive also, because (a, b) $\in R_1$, (b, c) $\in R_1$ imply that a $\geq$ b and b $\geq$ c which is turn imply that a $\geq$ c $\Rightarrow$ (a, c) $\in R_1$.

10. (B)

Sol. Given, *R* and *S* are relations on set *A*.

$\therefore R \subseteq A \times A$ and $S \subseteq A \times A$

$\Rightarrow R \cap C \subseteq A \times A$

$\Rightarrow R \cap S$ is also a relation on *A*.

Reflexivity : Let *a* be an arbitrary element of *A*. Then, $a \in A \Rightarrow (a, a) \in R$ and $(a, a) \in S$,

[$\because$ *R* and *S* are reflexive]

$\Rightarrow (a, a) \in R \cap S$

Thus, $(a, a) \in R \cap S$ for all $a \in A$.

So, $R \cap S$ is a reflexive relation on *A*.

Symmetry : Let $a, b \in A$ such that $(a, b) \in R \cap S$.

Then, $(a, b) \in R \cap S \Rightarrow (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$,

[$\because$ *R* and *S* are symmetric]

$\Rightarrow (b, a) \in R \cap S$

Thus, $(a,b) \in R \cap S$

$\Rightarrow (b,a) \in R \cap S$ for all $(a,b) \in R \cap S$.

So, $R \cap S$ is symmetric on *A*.

Transitivity : Let $a,b,c \in A$ such that $(a,b) \in R \cap S$ and $(b,c) \in R \cap S$. Then, $(a,b) \in R \cap S$ and $(b,c) \in R \cap S$

$\Rightarrow \{((a,b) \in R \text{ and} (a,b) \in S)\}$

and $\{((b,c) \in R \text{ and} (b,c) \in S\}$

$\Rightarrow \{(a,b) \in R, (b,c) \in R\}$ and $\{(a,b) \in S, (b,c) \in S\}$

$\Rightarrow (a,c) \in R$ and $(a,c) \in S$

$$\left[\begin{array}{l} \because R \text{ and } S \text{ are transitive So} \\ (a,b) \in R \text{ and } (b,c) \in R \Rightarrow (a,c) \in R \\ (a,b) \in S \text{ and } (b,c) \in S \Rightarrow (a,c) \in S \end{array}\right.$$

$\Rightarrow (a,c) \in R \cap S$

Thus, $(a,b) \in R \cap S$ and $(b,c) \in R \cap S \Rightarrow (a,c) \in R \cap S$.

So, $R \cap S$ is transitive on *A*.

Hence, *R* is an equivalence relation on *A*.

11. (C)

Sol. Here $R = \{(1,3),(2,2);(3,2)\}$, $S = \{(2,1);(3,2);(2,3)\}$

Then $RoS = \{(2,3);(3,2);(2,2)\}$.

12. (B)

Sol. Here $\alpha R \beta \Leftrightarrow \alpha \perp \beta \;\; \therefore \;\; \alpha \perp \beta \Leftrightarrow \beta \perp \alpha$

Hence, *R* is symmetric.

13. (D)

Sol. We have $(a,b)R(a,b)$ for all $(a,b) \in N \times N$

Since $a+b=b+a$. Hence, *R* is reflexive.

R is symmetric for we have $(a,b)R(c,d)$

$\Rightarrow a+d=b+c \quad \Rightarrow d+a=c+b$

$\Rightarrow c+b=d+a \Rightarrow (c,d)R(e,f)$.

Then by definition of *R*, we have

$a+d=b+c$ and $c+f=d+e$,

whence by addition, we get

$a+d+c+f=b+c+d+e$ or $a+f=b+e$

Hence, $(a,b)\ R(e,f)$

Thus, (*a*, *b*) $R(c,d)$ and $(c,d)R(e,f) \Rightarrow (a,b)R(e,\ f)$.

14. (C)

Sol. Here (3, 3), (6, 6), (9, 9), (12, 12), [Reflexive];

(3, 6), (6, 12), (3, 12), [Transitive].

Hence, reflexive and transitive only.

15. (C)

Sol. Given *A* = {1, 2, 3, 4}

R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)}

(2, 3) $\in$ *R* but (3, 2) $\notin$ *R*. Hence *R* is not symmetric.

R is not reflexive as (1, 1) $\notin$ *R*.

R is not a function as (2, 4) $\in$ *R* and (2, 3) $\in$ *R*.

R is not transitive as (1, 3) $\in$ *R* and (3, 1) $\in$ *R* but (1, 1) $\notin$ *R*.

16. (D)

Sol. Total number of reflexive relations in a set with *n* elements $= 2^n$.

Therefore, total number of reflexive relation set with 4 elements $= 2^4$.

17. (A)

Sol. Since $1 + a.a = 1 + a^2 > 0, \forall a \in S$, $\therefore$ $(a,a) \in R$

$\therefore$ *R* is reflexive.

Also $(a,b) \in R \;\Rightarrow 1+ab > 0 \;\Rightarrow 1+ba > 0$

$\Rightarrow (b,a) \in R$,

$\therefore$ *R* is symmetric.

$\because (a,b) \in R$ and $(b,c) \in R$ need not imply $(a,c) \in R$. Hence, *R* is not transitive.

18. (A)

Sol. $A = \{2,4,6\}$; $B = \{2,3,5\}$

$\therefore$ *A* $\times$ *B* contains $3 \times 3 = 9$ elements.

Hence, number of relations from *A* to *B* $= 2^9$.

19. (B)

Sol. 1. R is not symmetric so it is incorrect.

2. $S_1 \neq S_2$ so not reflexive

Let S_1 = {1, 2, 3} & S_2 = {1, 2}

it satisfies the condition

$S_1 \not\subset S_2 \Rightarrow S_2 \not\subset S_1$

So non symmetric.

let S_1 = {1, 2}, S_2 = {4, 5}, S_3 = {1, 2, 3}

as $S_1 \not\subset S_2$ and $S_1 \not\subset S_3$ S_1 S_3

so non transitive.

20. (A)

Sol. Let $A = \{1,2,3\}$ and *R* = {(1, 1), (1, 2)}, *S* = {(2, 2) (2, 3)} be transitive relations on *A*.

Then *R* $\cup$ *S* = {(1, 1); (1, 2); (2, 2); (2, 3)}

Obviously, *R* $\cup$ *S* is not transitive. Since (1, 2) $\in$ *R* $\cup$ *S* and $(2,3) \in R \cup S$ but (1, 3) $\notin R \cup S$.

21. 5

Sol. If $x \in A$, $y \in B$ then $x > y$, so R = {(3, 2), (4, 2), (5, 2), (6, 2), (6, 5)}

22. 4

Sol. As A = {1, 2}, B = {0}

$\therefore$ n(A) = 2, n(B) = 1

$\therefore$ number of relations from A to B is = $2^{n(A)\times n(B)}$

$= 2^{2\times 1} = 4$

23. 512

Sol. A = {2, 4, 6} $\Rightarrow$ n(1) = 3

$\therefore$ n(A × B) = 9

$\therefore$ no. of req. relations = $2^9 = 512$

B = {2, 3, 5} $\Rightarrow$ n(2) = 3

24. 0

Sol. y - x = x+ y only if x = 0 But $\notin$ N

25. 4

Sol. $n^2 = 16$

26. 1

Sol. need to be adjoined to make the relation transitive

27. 7

Sol. R is reflexive if it contains (1,1)(2,2)(3,3)

$\because$(1,2)∈R, (2,3)∈R

$\therefore$R is symmetric if (2,1),(3,2)∈R

Now,

R={(1,1),(2,2),(3,3),(2,1),(3,2),(2,3),(1,2)}

R will be transitive if (3,1);(1,3)∈R. Thus, R becomes and equivalence relation by adding (1,1)(2,2)(3,3)(2,1)(3,2)(1,3)(1,2). Hence, the total number of ordered pairs is 7.

28. 1

Sol. Here R is a relation A to B defined by 'x is greater then y' R={(2,1);(3,1)}.Hence, range of R={1}

29. 4

Sol. R = {(1,3), (1,5), (2,3), (2,5), (3,5), (4,5))

$\therefore R^{-1}$ = {(3,1), (5,1), (3,2), (5,2), (5,3), (5,4)}

$\therefore RoR^{-1}$ =(3,1)∈R^{-1} and(1,5)∈R

$\therefore$(3,5)∈RoR^{-1} etc.

RoR^{-1} = {(3,3), (3,5), (5,3), (5,5)}

30. 1

Sol. for any integer

| a – b | < 1 Þ | a – b | = 0

a = b.