JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS

CLASS :- 11th **PAPER CODE :- CWT-11**

CHAPTER :- COMPLEX NUMBER

ANSWER KEY

1.	(A)	**2.**	(A)	**3.**	(C)	**4.**	(C)	**5.**	(D)	**6.**	(B)	**7.**	(D)
8.	(A)	**9.**	(D)	**10.**	(B)	**11.**	(B)	**12.**	(A)	**13.**	(B)	**14.**	(C)
15.	(C)	**16.**	(A)	**17.**	(B)	**18.**	(A)	**19.**	(A)	**20.**	(C)	**21.**	4
22.	30	**23.**	7	**24.**	3	**25.**	2	**26.**	3	**27.**	9	**28.**	1
29.	1	**30.**	8										

SOLUTIONS

1. (A)

Sol. Let ; $a = r\cos\theta$, $b = r\sin\theta$

Then $(a + ib)^5 + (b + ia)^5$

$= r^5((\cos\theta + i\sin\theta)^5 + (\sin\theta + i\cos\theta)^5$

$$= r^5\left[(\cos 5\theta + i\sin 5\theta) + \left\{\cos\left(\frac{\pi}{2}-\theta\right) + i\sin\left(\frac{\pi}{2}-\theta\right)\right\}^5\right]$$

$$= r^5\left[(\cos 5\theta + i\sin 5\theta) + \cos 5\left(\frac{\pi}{2}-\theta\right) + i\sin 5\left(\frac{\pi}{2}-\theta\right)\right]$$

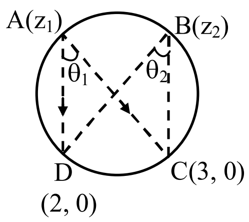
$= r^5[(\cos 5\theta + i\sin 5\theta)(1 + i)]$.

This is a complex number whose real and imaginary parts are equal. So locus of such a point will be $y = x$.

2. (A)

Sol. $\arg\left(\frac{3-z_1}{2-z_1}\right) + \arg\left(\frac{2-z_2}{3-z_2}\right)$

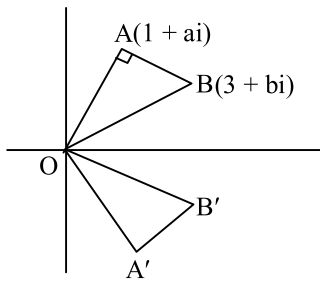
$= \arg\left(\frac{3-z_1}{2-z_1}\cdot\frac{2-z_2}{3-z_2}\right) = \theta_1 + \theta_2 = 0$



Now if, $\left(\frac{3-z_1}{2-z_1}\cdot\frac{2-z_2}{3-z_2}\right)$ is a positive real number, then its argument will be zero. So chord DC subtends equal angle at A and B. So points are concyclic for $k > 0$.

3. (C)

Sol. Since $\angle(OAB) = \frac{\pi}{2}$ and $OA = AB$,



$(3 + bi) - (1 + ai) = (-1-ai)i$

$2 + (b - a)i = a - i$

Comparison gives $a = 2$ and $b = 1$.

Another figure is also possible.

This gives $a = -2$ and $b = -1$.

4. (C)

Sol. Let $u = \frac{z-1}{e^{\theta i}}$ $\Rightarrow \frac{e^{\theta i}}{z-1} = \frac{1}{4}$.

Now $\left(u + \frac{1}{u}\right) - \left(\bar{u} + \frac{1}{\bar{u}}\right) = 0$

$\Rightarrow (u - \bar{u})\left(1 - \frac{1}{u\bar{u}}\right) = 0$

If u is not purely real, then $u\bar{u} = 1$

$\Rightarrow \left|\frac{z-1}{e^{\theta i}}\right| = 1 \Rightarrow |z - 1| = 1$

5. (D)

Sol. The given lines intersect, if the shortest distance between the lines is zero.

We know that the shortest distance between the lines $r = a_1 + (\lambda\vec{b}_1)$ and $r = a_2 + \lambda b_2$ is $\frac{|(a_1 - a_2).b_1 \times b_2|}{|b_1 \times b_2|}$

So the shortest distance between the given lines is zero if

$(i - j - (2i - j)\,.\,(2i + k) \times (i + j - k) = 0$

$$\text{L.H.S.} = \begin{vmatrix} -1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1 \neq 0$$

Hence the given lines do not intersect.

6. (B)

Sol. Equation of a plane passing through the line of intersection of the given planes is

$2x - y + 3z + 5 + \lambda(5x - 4y - 2z + 1) = 0$

or $(2 + 5\lambda)x - (1 + 4\lambda)y + (3 - 2\lambda)z + 5 + \lambda = 0$

This will be perpendicular to the plane $2x - y + 3z + 5 = 0$

if $2(2 + 5\lambda) + (1 + 4\lambda) + 3(3 - 2\lambda) = 0$

$\Rightarrow \lambda = -7/4$ and the required equation of the plane is

$4(2x - y + 3z + 5) - 7(5x - 4y - 2z + 1) = 0$

$\Rightarrow 27x - 24y - 26z - 13 = 0$

7. (D)

Sol. We have $|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right| \le \left|z - \frac{4}{z}\right| + \frac{4}{|z|}$

$= 2 + \frac{4}{|z|}$

$\Rightarrow |z|^2 \le 2|z| + 4 \quad \Rightarrow (|z| - 1)^2 \le 5$

$\Rightarrow |z| - 1 \le \sqrt{5} \Rightarrow |z| \le \sqrt{5} + 1$

Also, for $z = \sqrt{5} + 1$

$\left|z - \frac{4}{z}\right| = 2$

Therefore, the greatest value of $|z|$ is $\sqrt{5} + 1$.

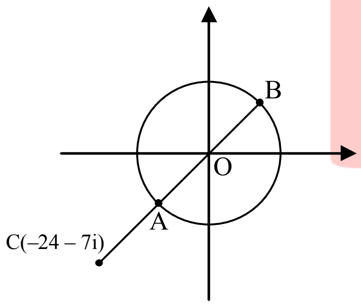
8. (A)

Sol. Note that $|z| = 6$ represents a circle. As $|z_2| = 6$,

$|z_1 + z_2| = |z_2 - (-24 - 7i)|$

represent distance between a point on the circle $|z| = 6$ and the point $C(-24 - 7i)$.

$|z_1 + z_2|$ will be greatest and least at points B and A which are the end points of the diameter of the cirlcle through C. As OC = 25, CA = OC – OA = 25 – 6 = 19 and CB = OC + OB = 25 + 6 = 31.



9. (D)

Sol. $z = \frac{\alpha + \beta t}{\gamma + \delta t} \Rightarrow (\gamma + \delta t) z = \alpha + \beta t$

$\Rightarrow (\delta z - \beta)t = \alpha - \gamma z$

$\Rightarrow t = \frac{\alpha - \gamma z}{\delta z - \beta} \qquad [\because \alpha\delta - \beta\gamma \ne 0]$

As t is real, $\frac{\alpha - \gamma z}{\delta z - \beta} = \frac{\bar{\alpha} - \bar{\gamma}\bar{z}}{\bar{\delta}\bar{z} - \bar{\beta}}$

$\Rightarrow (\alpha - \gamma z)(\bar{\delta}\bar{z} - \bar{\beta}) = (\bar{\alpha} - \bar{\gamma}\bar{z})(\delta z - \beta)$

$\Rightarrow (\bar{\gamma}\delta - \gamma\bar{\delta})z\bar{z} + (\gamma\bar{\beta} - \bar{\alpha}\delta)z + (\alpha\bar{\delta} - \beta\bar{\gamma})\bar{z}$

$= (\alpha\bar{\beta} - \bar{\alpha}\beta) \quad \ldots(1)$

Since $\frac{\gamma}{\delta}$ is real, $\frac{\gamma}{\delta} = \frac{\bar{\gamma}}{\bar{\delta}}$ or $\gamma\bar{\delta} - \delta\bar{\gamma} = 0$

Therefore (1) can be written as $\bar{a}z + a\bar{z} = c$ ...(2)

where $a = i(\alpha\bar{\delta} - \beta\bar{\gamma})$ and $c = i(\bar{\alpha}\beta - \alpha\bar{\beta})$

Note that $a \ne 0$ for if a = 0 then

$a\bar{\delta} - \beta\bar{\gamma} = 0 \Rightarrow \frac{\alpha}{\beta} = \frac{\bar{\gamma}}{\bar{\delta}} = \frac{\gamma}{\delta} \quad [\because \frac{\gamma}{\delta}$ is real]

$\Rightarrow \alpha\delta - \beta\gamma = 0$,

which is against hypothesis.

Also, note that $c = i(\bar{\alpha}\beta - \alpha\bar{\beta})$ is a purely real number.

Thus, $z = \frac{\alpha + \beta t}{\gamma + \delta t}$ represents a straight line.

10. (B)

Sol. $a\frac{(2x+1)^2}{(x-3)^2} + b\frac{(2x+1)}{(x-3)} + c = 0$

$\Rightarrow \frac{2x+1}{x-3} = \alpha$ or $\frac{2x+1}{x-3} = \beta$

$\Rightarrow 2x + 1 = \alpha x - 3\alpha$

$\Rightarrow x(\alpha - 2) = 1 + 3\alpha$

$\Rightarrow x = \frac{1+3\alpha}{\alpha - 2}, \frac{1+3\beta}{\beta - 2}$

11. (B)

Sol. $\arg(zw) = \pi \Rightarrow zw = \bar{z}.\bar{w} \Rightarrow zw.w = \bar{z}\bar{w}.w$

$\Rightarrow z.w^2 = \bar{z}.|w|^2 \Rightarrow zw^2 = \bar{z}|z|^2$

$\Rightarrow zw^2 = \bar{z}z\bar{z}$

$\Rightarrow w^2 = (\bar{z})^2$

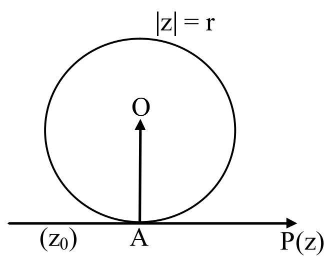
$\Rightarrow \bar{z} = w$ or $-w$

$\Rightarrow z = \bar{w}$ or $-\bar{w}$

but only $z = -\bar{w}$ satisfies $\arg(zw) = \pi$

12. (A)

Sol. $\frac{z - z_0}{z_0}$ is purely imaginary



$\frac{z - z_0}{z_0} + \frac{\bar{z} - \bar{z}_0}{\bar{z}_0} = 0$

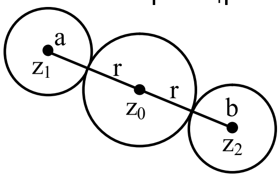
$\Rightarrow \frac{z}{z_0} + \overline{\left(\frac{z}{z_0}\right)} = 2$

$\Rightarrow 2R_e\left(\frac{z}{z_0}\right) = 2$

$\Rightarrow R_e\left(\frac{z}{z_0}\right) = 1$

13. (B)

Sol. Centre of the circle be z_0 and radius r. Then its equation is $|z - z_0| = r$ circle (1) touches the circle $|z - z_1| = a$ externally.



Distance between centres = sum of radii

$|z_0 - z_1| = a + r$ ...(1)

$|z_0 - z_2| = b + r$ ...(2)

(1) – (2)

$|z_0 - z_1| - |z_0 - z_2| = a - b$

$\therefore$ z_0 lies on the curve $|z - z_1| - |z - z_2| = a - b$ which is equation of a hyperbola.

14. (C)

Sol. $\frac{1 - ix}{1 + ix} = a - ib \Rightarrow 1 - ix = (a - ib)(1 + ix)$

$\Rightarrow 1 - ix = a + aix - ib + bx$

$\Rightarrow (1 - a + ib) = x\,[ai + b + i]$

$\Rightarrow x = \frac{(1-a) + ib}{b + i(a+1)} \times \frac{b - i(a+1)}{b - i(a+1)}$

$= \frac{b(1-a) + b(a+1) + i(b^2 - (1 - a^2))}{b^2 + (a+1)^2}$

x is real if $b^2 + a^2 - 1 = 0$

i.e. $a^2 + b^2 = 1$

15. (C)

Sol. Let the vertices of the square ABCD are 1, –1, i and –i in the Argand Diagram.

Let P be represented by z, then $\angle PAB + \angle PBC + \angle PCD + \angle PDA$

$= \arg\left(\frac{z^4 - 1}{4}\right) = \arg(z^4 - 1)$.

Since z^4 also lies inside the square

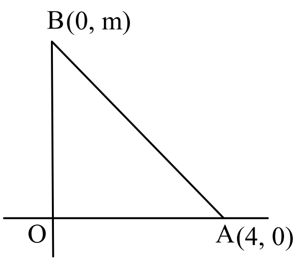
$\Rightarrow \frac{5\pi}{4} \ge \arg(z^4 - 1) \ge \frac{3\pi}{4}$

16. **(A)**

Sol. $|Z - mi| = m + 5$ represents a circle with mi or B(0, m) as centre and radius m + 5.

$|Z - 4| < 3$ represents the interior of a circle with centre 4 or A(4, 0) and radius 3.

If there is to be at least one Z satisfying both the two circles should intersect.



(i.e.) $r_1 \sim r_2 < d < r_1 + r_2$

$m + 5 - 3 < \sqrt{m^2 + 16} < m + 5 + 3$

squaring, $m^2 + 4m + 4 < m^2 + 16 < m^2 + 16m + 64$

$\therefore$ m < 3 and m > –3

$\therefore m \in (-3, 3)$

17. (B)

Sol. $Z^{\frac{1}{15}} = \cos\frac{2r\pi}{15} + i\sin\frac{2r\pi}{15}$ r = 0, 1, 2, ...14

$Z^{\frac{1}{25}} = \cos\frac{2m\pi}{25} + i\sin\frac{2m\pi}{25}$, m = 0, 1, 2, ...24

$\Rightarrow \frac{2r\pi}{15} = \frac{2m\pi}{25}$

$\Rightarrow \frac{r}{3} = \frac{m}{5} \Rightarrow$ r = 0, 3, 6, 9, 12 or m = 0, 5, 10, 15, 20

$\Rightarrow$ 5 common roots

18. (A)

Sol. $2\cos\theta = x + \frac{1}{x}$

$\Rightarrow x^2 - (2\cos\theta)x + 1 = 0$

$\Rightarrow x = \cos\theta \pm i\sin\theta$

Similarly, $y = \cos\phi \pm i\sin\phi$

taking $x = \cos\theta + i\sin\theta$ and $y = \cos\phi + i\sin\phi$

$\frac{x}{y} + \frac{y}{x} = \frac{\cos\theta + i\sin\theta}{\cos\phi + i\sin\phi} + \frac{\cos\phi + i\sin\phi}{\cos\theta + i\sin\theta}$

$= (\cos(\theta - \phi) + i\sin(\theta - \phi)) + \cos(\phi - \theta) + i\sin(\phi - \theta)$

$= \cos(\theta - \phi) + i\sin(\theta - \phi) + \cos(\theta - \phi) - i\sin(\theta - \phi)$

$= 2\cos(\theta - \phi)$

19. (A)

Sol. $|Z - 1| = 1 \Rightarrow Z - 1 = e^{i\theta}$

$\Rightarrow \frac{Z-2}{Z} = \frac{e^{i\theta} - 1}{e^{i\theta} + 1} = \frac{\cos\theta - 1 + i\sin\theta}{\cos\theta + 1 + i\sin\theta}$

$= \frac{2\sin\frac{\theta}{2}\left(i\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)} = i\tan\frac{\theta}{2}$

$= i\tan(\arg Z)$

$(\because \arg Z = \arg(1 + \cos\theta + i\sin\theta) = \arg\left(2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)\right) = \frac{\theta}{2})$

20. (C)

Sol. $\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}=0$

$\Rightarrow \sum \frac{1}{r_1(\cos\theta_1 + i\sin\theta_1)} = 0$

$\Rightarrow \sum \frac{\cos\theta_1 - i\sin\theta_1}{r_1} = 0$

$\Rightarrow \sum \frac{\cos\theta_1 + i\sin\theta_1}{r_1} = 0$

$\Rightarrow \sum \frac{(\cos\theta_1 + i\sin\theta_1)^2}{r_1(\cos\theta_1 + i\sin\theta_1)} = 0$

$\Rightarrow \sum \frac{\cos 2\theta_1 + i\sin 2\theta_1}{Z_1} = 0$

$\Rightarrow \frac{1}{3}\frac{(\cos 2\theta_1 + i\sin 2\theta_1)}{Z_1} = 0$

$\Rightarrow$ the centroid of the Δ ABC is the origin

$\Rightarrow$ origin lies inside the Δ.

21. 4

Sol. $zi - \bar{z}i + 2 = 0$

$\Rightarrow (z - \bar{z}) = 2i \Rightarrow \text{Im}(z) = 1$

$z(1+i) + \bar{z}(1-i) + 2 = 0$

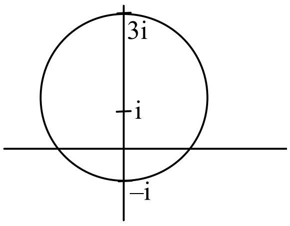
$\Rightarrow (z + \bar{z}) + i(z - \bar{z}) + 2 = 0$

$\Rightarrow z = -\bar{z} \Rightarrow z = i$

$\Downarrow$

Re(z) = 0

Let the point on the line be z so |z –i| = 2



$|z|_{max} = |3i| = 3$

$|z|_{min} = |-i| = 1$

sum = 4

22. 30

Sol. $|z| + |z-1| + |2z-3|$

$= |z| + |z-1| + |3-2z| \geq |z + z - 1 + 3 - 2z| = 2$

$\therefore |z| + |z-1| + |2z-3| \geq 2$

$\therefore \lambda = 2$

then $2[x] + 3 = 3[x - \lambda]$

$= 3[x-2]$

$\Rightarrow 2[x] + 3 = 3([x] - 2)$

or $[x] = 9$, then $y = 2.9 + 3 = 21$

$\therefore [x + y] = [x + 21] = [x] + 21 = 9 + 21 = 30$

23. 7

Sol. $\because f(x) = A_0 + \sum_{k=1}^{20} A_k x^k = \sum_{k=0}^{20} A_k x^k$

$\because \sum_{r=0}^{6} f(\alpha^r x) = f(x) + f(\alpha x) + f(\alpha^2 x) + f(\alpha^3 x) + f(\alpha^4 x) + f(\alpha^5 x) + f(\alpha^6 x)$

$= \sum_{k=0}^{20} (A_k x^k + A_k(\alpha x)^k + A_k(\alpha^2 x)^k + A_k(\alpha^3 x)^k + A_k(\alpha^4 x)^k + A_k(\alpha^5 x)^k + A_k(\alpha^6 x)^k)$

$= \sum_{k=0}^{20} A_k x^k (1 + (\alpha)^k (\alpha^2)^k + (\alpha^3)^k + (\alpha^4)^k + (\alpha^5)^k + (\alpha^6)^k)$

$= A_0 x^0 (7) + A_7 x^7 (7) + A_{14} x^{14} (7)$

($\because$ sum of 7, 7th roots of unity is zero ($\because \alpha^{2\pi i/7} = 1$))

$= 7(A_0 + A_7 x^7 + A_{14} x^{14})$

$\therefore$ n = 7

24. 3

Sol. $(z_1 - 3z_2)(\bar{z}_1 - 3\bar{z}_2) = (3 - z_1\bar{z}_2)(3 - \bar{z}_1 z_2)$

$\Rightarrow |z_1|^2 - |z_1|^2 |z_2|^2 + 9|z_2|^2 - 9 = 0$

$\Rightarrow (1 - |z_2|^2)(|z_1|^2 - 9) = 0$

$\Rightarrow |z_1| = 3$

25. 2

Sol. $z = \frac{z_1^8}{\bar{z}_2^6} + \frac{z_2^6}{\bar{z}_1^8} = \frac{|z_1|^{16} + |z_2|^{12}}{|\bar{z}_2|^6 (\bar{z}_1)^8}$

$\arg(z) = -8\arg(\bar{z}_1) - 6\arg(\bar{z}_2) + 2k\pi$, $k \in I$

$= 8\arg(z_1) + 6\arg(z_2) + 2k\pi$

$= 8.\frac{\pi}{6} + 6.\frac{\pi}{4} + 2k\pi = \frac{4\pi}{3} + \frac{3\pi}{2} - 2\pi = \frac{5\pi}{6}$

$\therefore 4\sin\theta = 4\sin\frac{5\pi}{6} = 2$

26. 3

Sol. Let z = x + iy

$x + iy + \sqrt{2}|x + 1 + iy| + i = 0$

$x + \sqrt{2}\sqrt{(x+1)^2 + y^2} + i(y+1) = 0$

$x + \sqrt{2}\sqrt{(x+1)^2 + y^2} = 0$ & $y + 1 = 0 \Rightarrow y = -1$

So $x + \sqrt{2}\sqrt{(x+1)^2 + 1} = 0$

$2((x+1)^2 + 1) = x^2 \Rightarrow x^2 + 4x + 4 = 0 \Rightarrow x = -2$

$\therefore z_1 = -2 - i$

Now $e^{iz_1} = e^{i(-2-i)} = e^{1-2i}$

Real part of $e^{iz_1} = e\cos 2 = e^a \cos b$

$\therefore a + b = 1 + 2 = 3$

27. 9

Sol. $|z| = 2 \Rightarrow$ any $z = 2\cos\theta + i\,2\sin\theta$

$$|w| = \frac{|z+1|}{|z-1|} = \frac{\sqrt{(2\cos\theta+1)^2+4\sin^2\theta}}{\sqrt{(2\cos\theta-1)^2+4\sin^2\theta}}$$

$$= \frac{\sqrt{5+4\cos\theta}}{\sqrt{5-4\cos\theta}}$$

$|W|_{max} = 3$, $|W|_{min} = \frac{1}{3}$

$$\frac{M}{m} = 9$$

28. 1

Sol. $(x_1\, x_2\, x_3 \ldots\ldots \infty)^2\,(z_1\, z_2\, z_3 \ldots\ldots \infty)^4$

$$\left[\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2}+i\sin\frac{\pi}{2^2}\right)\left(\cos\frac{\pi}{2^3}+i\sin\frac{\pi}{2^3}\right)\ldots\infty\right]^2$$

$$\left[\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{3^2}+i\sin\frac{\pi}{3^2}\right)+\ldots\infty\right]^4$$

$$= \left[\cos\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\frac{\pi}{2^3}+\ldots\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\frac{\pi}{2^3}+\ldots\right)\right]^2$$

$$\cdot\left[\cos\left(\frac{\pi}{3}+\frac{\pi}{3^2}+\frac{\pi}{3^3}+\ldots\right)+i\sin\left(\frac{\pi}{3}+\frac{\pi}{3^2}+\frac{\pi}{3^3}+\ldots\right)\right]^4$$

$$= \left[\cos\left(\frac{\pi/2}{1-\frac{1}{2}}\right)+i\sin\left(\frac{\pi/2}{1-\frac{1}{2}}\right)\right]^2$$

$$\left[\cos\left(\frac{\pi/3}{1-\frac{1}{3}}\right)+i\sin\left(\frac{\pi/3}{1-\frac{1}{3}}\right)\right]^4$$

$= (\cos\pi + i\sin\pi)^2\,(\cos\pi/2 + i\sin\pi/2)^4 = (-1)^2\,(i)^4 = 1.$

29. 1

Sol. $\frac{a}{b}+\frac{b}{c}+\frac{c}{a} = 1$

$\Rightarrow \frac{\text{cis}\,\alpha}{\text{cis}\,\beta}+\frac{\text{cis}\,\beta}{\text{cis}\,\gamma}+\frac{\text{cis}\,\gamma}{\text{cis}\,\alpha} = 1,$

Where cis θ represents $\cos\theta + i\sin\theta$

cis $(\alpha-\beta)$ + cis $(\beta-\gamma)$ + cis $(\gamma-\alpha) = 1$

Equation real parts of both sides

$\Rightarrow \cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha) = 1$

30. 8

Sol. $x^2 - x + 1 = 0$

$\Rightarrow x = \frac{1\pm\sqrt{3}i}{2} = -\omega, -\omega^2$

$\therefore \sum_{n=1}^{5}\left(x^{2n}+\frac{1}{x^{2n}}+2\right)$

$\Rightarrow \left(x^2+\frac{1}{x^2}+2\right) + \left(x^4+\frac{1}{x^4}+2\right) + \left(x^6+\frac{1}{x^6}+2\right) + \left(x^8+\frac{1}{x^8}+2\right) + \left(x^{10}+\frac{1}{x^{10}}+2\right)$

$\Rightarrow (\omega^2 + \omega^4 + \omega^6 + \omega^8 + \omega^{10}) + \left(\frac{1}{\omega^2}+\frac{1}{\omega^4}+\frac{1}{\omega^6}+\frac{1}{\omega^8}+\frac{1}{\omega^{10}}\right) + 10$

$\Rightarrow -1-1+10 = 8.$