JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 11th **PAPER CODE :- CWT-10**
CHAPTER :- CONIC SECTION

ANSWER KEY

1.	(C)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(C)	**6.**	(A)	**7.**	(B)
8.	(C)	**9.**	(C)	**10.**	(A)	**11.**	(A)	**12.**	(A)	**13.**	(C)	**14.**	(D)
15.	(C)	**16.**	(C)	**17.**	(C)	**18.**	(C)	**19.**	(B)	**20.**	(A)	**21.**	50
22.	186	**23.**	8	**24.**	10	**25.**	6	**26.**	2	**27.**	14	**28.**	23
29.	10	**30.**	6										

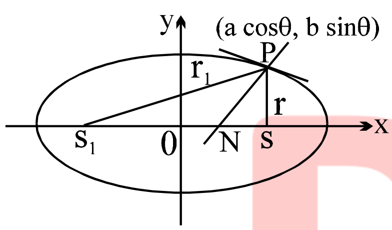
SOLUTIONS

1. (C)

Sol. Equation of normal $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 e^2$

put $y = 0$ $x = a e^2 \cos\theta$

$\Rightarrow$ N $(a e^2 \cos\theta, 0)$



now $(PN)^2 = (a \cos\theta - a e^2 \cos\theta)^2 + b^2 \sin^2\theta$

$= a^2 \cos^2\theta (1 - e^2)^2 + b^2 \sin^2\theta$

$= a^2 \cos^2\theta . \frac{b^4}{a^4} + b^2 \sin^2\theta$

$= \frac{b^4 \cos^2\theta + a^2 b^2 \sin^2\theta}{a^2}$

$l^2 = \frac{b^2}{a^2} [a^2 \sin^2\theta + b^2 \cos^2\theta]$ ....(1)

Also $(SP) \cdot (S_1P) = r r_1 = e (a/e - a \cos\theta). e (a/e + a \cos\theta)$

$= (a - e a \cos\theta) (a + ae \cos\theta) = a^2 - a^2 e^2 \cos^2\theta$

$= a^2 - (a^2 - b^2) \cos^2\theta \Rightarrow r r_1 = a^2 \sin^2\theta + b^2 \cos^2\theta$ ....(2)

From (1) and (2) $l = \frac{b}{a}\sqrt{r r_1}$

Note: For objective take point P as (0, b) or take a = b]

2. (B)

Sol. The equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(a \cos\theta, b \cos\theta)$ is $ax \sec\theta - by \operatorname{cosec}\theta = a^2 - b^2$

Given, ellipse equation as $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Length of major axis , a = 5 & length of minor axis, b = 3

$\therefore$ The required equation of normal is $5x \sec\theta - 3y \operatorname{cosec}\theta = 25 - 9$

$\Rightarrow 5x \sec\theta - 3y \operatorname{cosec}\theta = 16$...............(1)

Given normal equation $5x - 3y - = 8\sqrt{2}$

Multiplying both sides with $\sqrt{2}$

$5\sqrt{2}x - 3\sqrt{2}y = 16$........(2)

Comparing equation (1) & (2)

$\sec\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}$

3. (B)

Sol. The equation of the tangent is $\frac{x}{a} \cdot \left(\frac{1}{2}\right) + \frac{y}{b}\left(\frac{\sqrt{3}}{2}\right) = 1$...........(i)

Auxiliary circle is $x^2 + y^2 = a^2$................(ii)

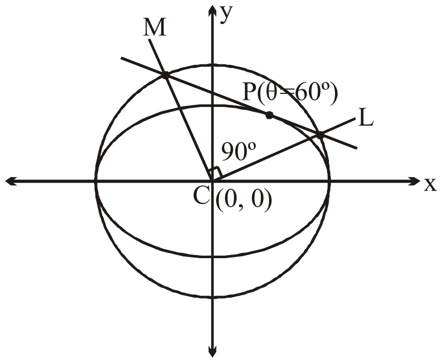
C is the centre.

Combined equation of CL, CM is obtained by homgenising (ii) with (i), i.e.,

$x^2 + y^2 - a^2 \left(\frac{x}{2a} + \frac{\sqrt{3}y}{2b}\right)^2 = 0$

Since $\angle LCM = 90°$

$\Rightarrow 1 - \frac{1}{4} + 1 - \frac{3a^2}{4b^2} = 0 \Rightarrow \frac{3a^2}{4b^2} = \frac{7}{4}$



$\Rightarrow 7b^2 = 3a^2 \Rightarrow 7a^2(1-e^2) = 3a^2$

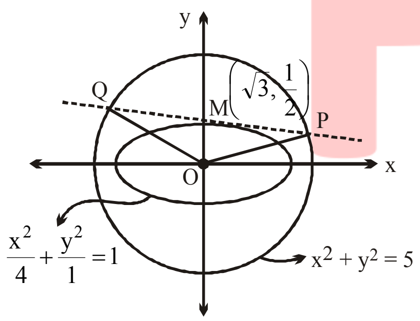
Hence $e = \frac{2}{\sqrt{7}}$ Ans.]

4. (A)

Sol. Equation of tangent at M $\left(\sqrt{3}, \frac{1}{2}\right)$ is

$\sqrt{3}\, x + 2y = 4$

Also, distance of PQ from (0, 0) = $\frac{4}{\sqrt{7}}$



Now, PQ = $2\sqrt{5 - \frac{16}{7}} = 2\sqrt{\frac{19}{7}}$

$\therefore$ Area of (ΔOPQ) = $\frac{1}{2} \times \frac{4}{\sqrt{7}} \times 2\sqrt{\frac{19}{7}}$

= $\frac{4\sqrt{19}}{7}$. **Ans.**]

5. (C)

Sol. Equation of tangent with slope = $\frac{-3}{4}$, is

$y = \frac{-3}{4}x + C$

Now, $C = \sqrt{32 \times \left(\frac{-3}{4}\right)^2 + 18} = \sqrt{18 + 18}$

= 6 (Using condition of tangency)

$\therefore\quad y = \frac{-3}{4}x + 6 \Rightarrow 3x + 4y = 24$

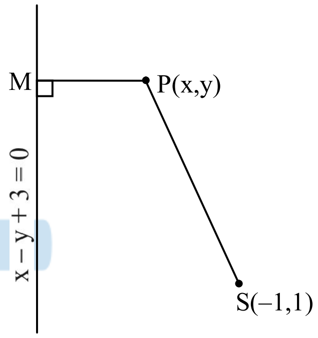
It meets the coordinate axes in A and B. So A(8, 0) and B(0, 6).

Hence, required area of ΔAOB = $\frac{1}{2}(8)(6)$

= 24. **Ans.**]

6. (A)

Sol. Let P (x, y) be any point on the ellipse whose focus is S (–1,1) and the directrix is $x - y + 3 = 0$.



PM is perpendicular from P (x, y) on the directrix $x - y + 3 = 0$.

Then by definition

SP = ePM

$\Rightarrow (SP)^2 = e^2 (PM)^2$

$\Rightarrow (x+1)^2 + (y-1)^2 = \frac{1}{4}\left\{\frac{x-y+3}{\sqrt{2}}\right\}^2$

$\Rightarrow 8(x^2 + y^2 + 2x - 2y + 2)$

$= x^2 + y^2 + 9 - 2xy + 6x - 6y$

$\Rightarrow 7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

which is the required equation of the ellipse.

7. (B)

Sol. Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Then coordinates of foci are ($\pm$ ae, 0).

$\therefore\quad ae = 2 \Rightarrow a \times \frac{1}{2} = 2$

$\left[\because e = \frac{1}{2}\right]$

$\Rightarrow\quad a = 4$

We have $b^2 = a^2(1 - e^2)$

$\therefore\quad b^2 = 16\left(1 - \frac{1}{4}\right) = 12$

Thus, the equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

8. (C)

Sol. SS' = 2ae, where a and e are length of semi-major axis and eccentricity respectively.

$\therefore \quad \sqrt{(9-3)^2+(12-4)^2} = 2ae$

$\therefore$ ae = 5

$\therefore$ centre is mid-point of SS'

$\therefore$ centre $\equiv$ (6, 8)

Let the equation of auxiliary circle be

$(x-6)^2 + (y-8)^2 = a^2$

We know that the foot of the perpendicular from the focus on any tangent lies on the auxiliary circle

$\therefore$ (1, – 4) lies on auxiliary circle

i.e. $(1-6)^2 + (-4-8)^2 = a^2 \Rightarrow a = 13$

$\because$ ae = 5 $\Rightarrow$ e = 5/13 Ans.]

9. (C)

Sol. Put $y = \frac{k}{x}$ in $x^2 + y^2 = k^2$, we get

$x^2 + \frac{k^2}{x^2} = k^2 \Rightarrow x^4 - k^2x^2 + k^2 = 0$

$\therefore \quad x^2 = \frac{k^2 \pm \sqrt{k^4 - 4k^2}}{2}$

For no intersection $k^2(k^2-4) < 0\,(k \neq 0)$

$\therefore$ $k \in (-2, 2) - \{0\}$

Hence integral values of k are {–1, 1}]

10. (A)

Sol. Chord of the hypererbola $x^2 - y^2 = a^2$ with middle point (h, k), is

$hx - ky = h^2 - k^2$

$\Rightarrow y = \frac{h}{k}x + \frac{k^2-h^2}{k}$ ....(1)

$\left(m = \frac{h}{k};\ c = \frac{k^2-h^2}{k}\right)$

As (1) touches $y^2 = 4ax$, condition of tangency gives

$c = \frac{a}{m} \Rightarrow \frac{k^2-h^2}{k} = \frac{ak}{h}$

$\Rightarrow x(y^2 - x^2) = 3y^2 \Rightarrow x^3 = y^2(x-3)$ **Ans.**]

11. (A)

Sol. Any tangent to given hypererbola, is

$\frac{x \cdot 2\sec\theta}{4} - \frac{y \cdot \tan\theta}{1} = 1$ ....(1)

Let (x_1, y_1) be the middle point of the chord of ellipse.

$\therefore$ Its equation is $\frac{xx_1}{4} + \frac{yy_1}{1} = \frac{x_1^2}{4} + \frac{y_1^2}{1}$ ....(2)

As (1) and (2) are identical, so

$\frac{2\sec\theta}{x_1} = \frac{-\tan\theta}{y_1} = \frac{1}{\frac{x_1^2}{4}+\frac{y_1^2}{1}} \Rightarrow \sec\theta$

$= \frac{2x_1}{x_1^2+4y_1^2}$ and $\tan\theta = \frac{-4y_1}{x_1^2+4y_1^2}$

As, $\sec^2\theta - \tan^2\theta = 1$,

so $\frac{4{x_1}^2}{\left(x_1^2+4y_1^2\right)^2} - \frac{16y_1^2}{\left(x_1^2+4y_1^2\right)^2} = 1$

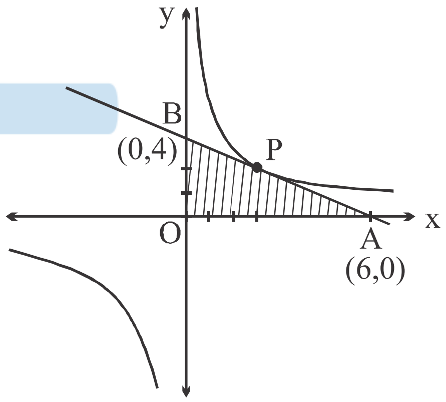
$\Rightarrow$ Locus is $(x^2 + 4y^2)^2 = 4(x^2 - 4y^2)$. **Ans.**]

12. (A)

Sol. Since L is the tangent to the hyperbola, we know that its slope is given by

$m = \frac{dy}{dx} = \frac{-6}{x^2}$ at x = 3, that is $m = \frac{-2}{3}$.

An Equation for this line is $y - 2 = \frac{-2}{3}(x-3)$.



From this equation we get that B = 4 and A = 6 which implies that the area is 12.]

13. (C)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $x^2 = cy$

$(2\sqrt{2}, 4)$ satisfy both curves

$\frac{8}{a^2} - \frac{16}{b^2} = 1$, 8 = c.4 $\Rightarrow$ c = 2

$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$

$$y'\big|_{(2\sqrt{2},\,4)} = \frac{b^2}{a^2}\cdot\frac{2\sqrt{2}}{4} = \frac{b^2}{\sqrt{2}a^2}$$

$$\Rightarrow \frac{b^2}{\sqrt{2}a^2} = 2\sqrt{2} \Rightarrow b^2 = 4a^2$$

2x = 2y' $\Rightarrow$ $\left.\frac{dy}{dx}\right|_{(2\sqrt{2},4)} = 2\sqrt{2}$

$\therefore$ $\frac{8}{a^2} - \frac{16}{4a^2} = 1$ $\Rightarrow$ $a^2 = 4$, $b^2 = 16$

$a^2 + b^2 + c = 16 + 14 + 2 = 22$. **Ans.**]

14. (D)

Sol. 2ae = SS'

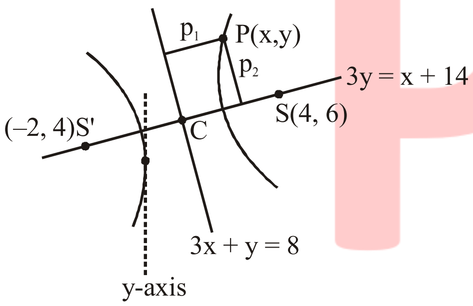
$\therefore$ ae = $\sqrt{10}$ ...... (1)

and product of distance from focus on y-axis gives b^2

$\therefore$ $b^2 = 8$ ...... (2)

$\therefore$ $a^2 = 2$, [$\because$ $b^2 + a^2 = a^2e^2$]

$\therefore$ Equation of hyperbola is



$$\frac{p_1^2}{a^2} - \frac{p_2^2}{b^2} = 1$$

$$\Rightarrow \frac{(3x+y-8)^2}{(\sqrt{10})^2\cdot 2} - \frac{(x-3y+14)^2}{(\sqrt{10})^2\cdot 8} = 1$$

i.e. $$\frac{(3x+y-8)^2}{20} - \frac{(x-3y+14)^2}{80} = 1$$

Ans.]

15. (C)

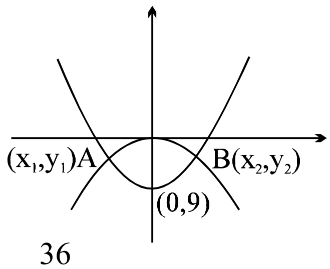
Sol. solving $x^2 - 9 = kx^2$

$x^2(k-1) + 0.x + 9 = 0$

$x_1 + x_2 = 0$ & $x_1 x_2 = \frac{9}{k-1}$

now, $|x_1 - x_2| = 10$

$$= \sqrt{(x_1+x_2)^2 - 4x_1x_2}$$



$$100 = \frac{36}{1-k}$$

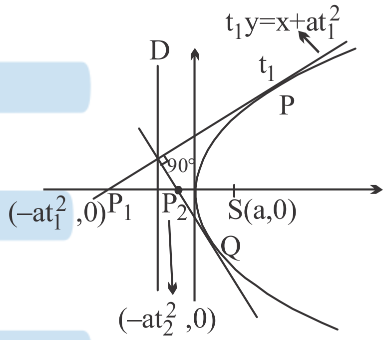
$100 - 100k = 36$ $\Rightarrow$ $k = \frac{64}{100} = \frac{16}{25}$ Ans]

16. (C)

Sol. $SP_1 = a(1 + t_1^2)$; $SP_2 = a(1 + t_2^2)$

also $t_1t_2 = -1$

$$\frac{1}{SP_1} = \frac{1}{a(1+t^2)} \;;\; \frac{1}{SP_2} = \frac{t^2}{a(1+t^2)}$$



$$\therefore \quad \frac{1}{SP_1} + \frac{1}{SP_2} = \frac{1}{a}$$ Ans.]

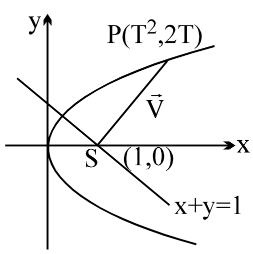
17. (C)

Sol. $\vec{V} = (T^2 - 1)\hat{i} + 2T\hat{j}$

$\vec{n} = \hat{j} - \hat{i}$

projection of $\vec{V}$ on $\vec{n}$

$$y = \frac{\vec{V}\cdot\vec{n}}{|\vec{n}|} = \frac{(1-T^2)+2T}{\sqrt{2}}$$



$\sqrt{2}\,y = 1 - T^2 + 2T$; $\sqrt{2}\dfrac{dy}{dt}$

$= -2T\dfrac{dT}{dt} + 2\dfrac{dT}{dt}$

Given $\dfrac{dx}{dt} = 4$; but $x = T^2$;

$\dfrac{dx}{dt} = 2T\dfrac{dT}{dt}$

when P(4, 4) then T = 2

$\Rightarrow \quad 4 = 2 \cdot 2\dfrac{dT}{dt}$; $\dfrac{dT}{dt} = 1$

$\therefore \quad \sqrt{2}\dfrac{dy}{dt} = -4 + 2 = -2$

$\Rightarrow \quad \dfrac{dy}{dt} = -\sqrt{2}$]

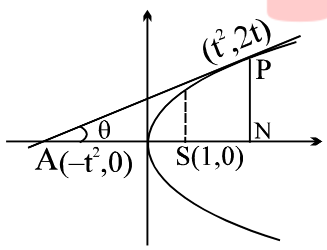
18. (C)

Sol. $A = \dfrac{(y_1^2 - 4ax_1)^{3/2}}{2a}$; $x_1 = 4$; $y_1 = 6$; $a = 2$]

19. (B)

Sol. T : $ty = x + t^2$, $\tan\theta = \dfrac{1}{t}$

$A = \dfrac{1}{2}$ (AN) (PN) $= \dfrac{1}{2}(2t^2)(2t)$



$A = 2t^3 = 2(t^2)^{3/2}$

i.e. $t^2 \in [1,4]$ & A_{max} occurs when $t^2 = 4 \Rightarrow A_{max} = 16$]

20. (A)

Sol. $y = x^2 - 2$; $x^2 + y^2 = 8$

$x^2 = y + 2$; $y^2 + y + 2 = 8$

$y^2 + y - 6 = 0 \Rightarrow y^2 + 3y - 2y - 6 = 0$

$\Rightarrow (y + 3)(y - 2) = 0 \Rightarrow y = 2$ or $y = -3$ (rejected)

If $y = 2$; $x = 2$ or -2

$\therefore$ B (2, 2) ; C = (–2, 2) and A (0, – 2)

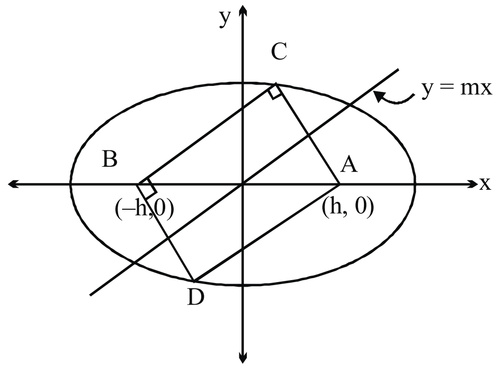
$\Rightarrow$ Area of $\Delta ABC = \dfrac{4 \times 4}{2} = 8$. **Ans.**]

21. 50

Sol. Image of A(h, 0) in the line mirror $mx - y = 0$

$$\frac{x-h}{m} = \frac{y-0}{-1} = -2\left(\frac{mh}{m^2+1}\right)$$

$$x = \frac{h(1-m^2)}{1+m^2},\quad y = \frac{2mh}{m^2+1}$$



$$\therefore\ C \equiv \left(\frac{h(1-m^2)}{1+m^2}, \frac{2mh}{m^2+1}\right)$$

Similarly $D \equiv \left(\dfrac{-h(1-m^2)}{1+m^2}, \dfrac{-2mh}{m^2+1}\right)$

$$AC = \sqrt{\left(h - \frac{h(1-m^2)}{1+m^2}\right)^2 + \frac{4m^2h^2}{(1+m^2)^2}}$$

$$= \sqrt{\frac{4m^4h^2}{(1+m^2)^2} + \frac{4m^2h^2}{(1+m^2)^2}} = \frac{2mh\sqrt{1+m^2}}{(1+m^2)}$$

$$AD = \sqrt{\left(h + \frac{h(1-m^2)}{1+m^2}\right)^2 + \frac{4m^2h^2}{(1+m^2)^2}}$$

$$= \sqrt{\frac{4h^2}{(1+m^2)^2} + \frac{4m^2h^2}{(1+m^2)^2}}$$

$$= \frac{2h}{(1+m^2)}\sqrt{1+m^2}$$

$$\therefore \text{Area} = AC \cdot AD = \frac{4mh^2(1+m^2)}{(1+m^2)^2}$$

$$= \frac{4h^2}{m + \dfrac{1}{m}}$$

$\text{Area}|_{max} = \dfrac{4 \times 25}{2} = 50$ **Ans.**]

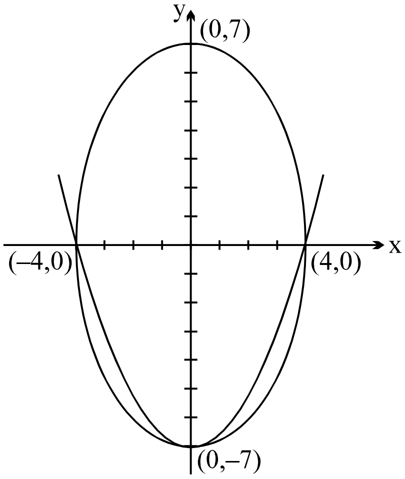
22. 186

Sol. Let $x - 3 = X$ and $y + 4 = Y$

hence the equation of the ellipse is

$\frac{X^2}{16} + \frac{Y^2}{49} = 1$

equation of a parabola with vertex on A(0, –7) can be taken as



$X^2 = \lambda(Y + 7)$

It passes through (4, 0)

$\therefore \quad 16 = 7\lambda \Rightarrow \quad \lambda = \frac{16}{7}$

Hence the equation of the parabola is new coordinate system is

$7X^2 = 16(Y + 7)$

put $X = x - 3$ and $Y = y + 4$

$7(x - 3)^2 = 16(y + 11)$

$\therefore \quad y = \frac{7}{16}(x - 3)^2 - 11$ or $16y$

$= 7(x - 3)^2 - 176$

compare it with $16y = a(x - h)^2 - k$

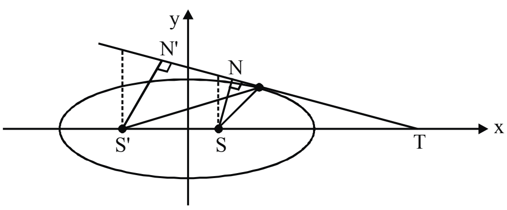
$a + h + k = 7 + 3 + 176 = 186$ Ans.]

23. 8

Sol. Area of (ΔPTS) × Area (ΔPTS')

$= \left(\frac{1}{2} \cdot PT \cdot SN\right) \times \frac{1}{2} \cdot PT \cdot S'N'$

$= \frac{(PT)^2 \cdot b^2}{4}$



$\therefore$ Aoording to satisfies

$\sum_{i=1}^{n} \frac{\text{area}(\Delta P_iT_iS) \cdot \text{area}(\Delta P_iT_iS')}{(P_iT_i)^2} = \frac{1}{4} \cdot 9 \cdot n$

$= 18$

n = 8 **Ans.**]

24. 10

Sol. Let $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $H : \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$; $e_E^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{3}{4}$

So, $a^2 = \frac{28}{3}$ and $b^2 = 7$

$\Rightarrow E : \frac{x^2}{\frac{28}{3}} + \frac{y^2}{7} = 1$

Now, slope of tangent at M (2, 2) on ellipse

$= \frac{-3}{4}$

So, slope of tangent at M (2, 2) on hyperbola

$\Rightarrow \frac{B^2}{A^2} = \frac{4}{3}$

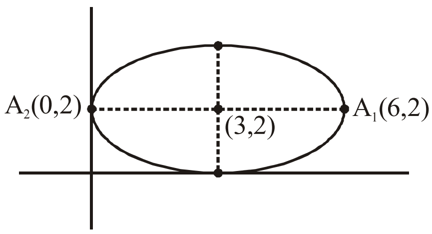
As, $e_h^2 = 1 + \frac{B^2}{A^2} \Rightarrow e_h^2 = 1 + \frac{4}{3} = \frac{7}{3}$

So, $e_h = \sqrt{\frac{7}{3}}$.

25. **6**

Sol. $C_1 : \frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} = 1$

$A_1 \equiv (6, 2)$ and $A_2 \equiv (0, 2)$



$|PA_1 - PA_2| = 3\sqrt{2}$.

Clearly, locus of P is a hyperbola whose A_1A_2 = 2ae = 6 foci are A_1 and A_2 and 2a = $3\sqrt{2}$.

e = $\sqrt{2}$

$\therefore$ Locus of P is a rectangular hyperbola

$a = b = \frac{3}{\sqrt{2}}$.

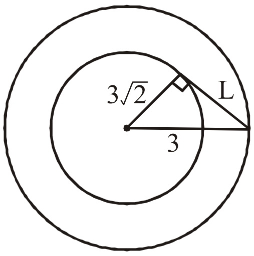
Equation of conic C_2 is $(x - 3)^2 - (y - 2)^2 = \frac{9}{2}$.

D_1 : Distance between foci of the conic is $A_1A_2 = 6$.

D_2 : A_1, A_2 are the foci of the conic C_2

$\therefore$ Product of the perpendicular from A_1 and A_2 upon its any tangent is equal to b^2 i.e., $\frac{9}{2}$.

D_3 : $L = \sqrt{9 - \frac{9}{2}} = \frac{3}{\sqrt{2}}$.



Hence $\left(\frac{D_1D_2}{D_3^{\,2}}\right) = 6$. **Ans.**]

26. 2

Sol. The equation of tangent at P(x, y) is

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

It passes through (0, –b), so $0 + \frac{y_1}{b} = 1$

$\Rightarrow y_1 = b$

Also, equation of normal at $P(x_1, y)$ is

$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2e^2$

It passes through $\left(2\sqrt{2}\,a, 0\right)$, so

$$\frac{a^2\left(2\sqrt{2}a\right)}{x_1} = a^2e^2$$

$\Rightarrow \quad x_1 = \frac{2\sqrt{2}\,a}{e^2}$

So, $P\left(x_1 = \frac{2\sqrt{2}\,a}{e^2},\ y_1 = b\right)$

As $P(x_1, y_1)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = 1, so $\frac{8a^2}{e^4a^2} - \frac{b^2}{b^2} = 1$

$\Rightarrow \quad \frac{8}{e^4} = 2 \Rightarrow e^4 = 4 \Rightarrow e^2 = 2$

Ans.]

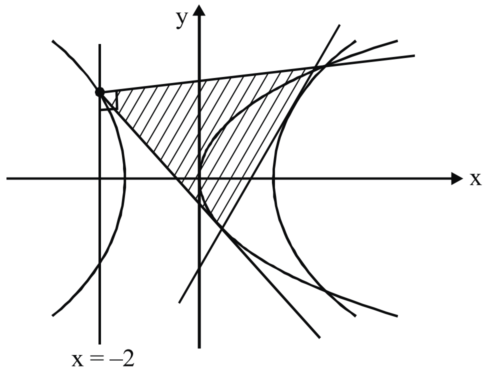
27. 14

Sol. $C_1 : |z - 2| = \left|\frac{z + \bar{z} + 4}{2}\right|$

$\Rightarrow C_1 : y^2 = 8x$ (parabola)

$C_2 : \left|\left|z + \sqrt{5}\right| - \left|z - \sqrt{5}\right|\right| = 2$

$\Rightarrow C_2 : \frac{x^2}{1} - \frac{y^2}{4} = 1$ (hyperbola)



Point on the hyperbola from where two perpendicular tangents are drawn also lies on the directrix of the parabola

$\therefore \quad x = -2, \quad \frac{4}{1} - \frac{y^2}{4} = 1 \Rightarrow y = \pm 2\sqrt{3}$

$\left(-2, \pm 2\sqrt{3}\right)$

Required area = $\frac{S_1^{3/2}}{2|a|} = \frac{(12 - 8(-2))^{3/2}}{2 \cdot 2}$

$= \frac{28 \cdot 2\sqrt{7}}{4} = 14\sqrt{7}$ **Ans.**]

28. 23

Sol. $(y - 5)^2 = 8(x - 1)$

Focus of the parabola is (3, 5)

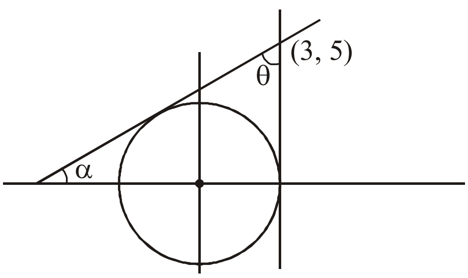
Equation of tangent to the $x^2 + y^2 = 9$, from the point (3, 5)

$y - 5 = m(x - 3)$

$mx - y + 5 - 3m = 0$

Applying p = r

$\left|\frac{5 - 3m}{\sqrt{1 + m^2}}\right| = 3 \Rightarrow m = \frac{8}{15}$ or ∞



$\therefore$ $\tan\alpha = \frac{8}{15}$

$\Rightarrow \theta = \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \tan^{-1}\frac{8}{15} = \cot^{-1}\left(\frac{8}{15}\right) = \tan^{-1}\left(\frac{15}{8}\right)$

$\therefore$ a + b = 15 + 8 = 23 **Ans.**]

29. 10

Sol. Parabola : $(y-1)^2 = 4(x-4)$, $\lambda = 4$

tangent to parabola is $y - 1 = m(x-4) + \frac{1}{m}$

it passes through (8, 5) $\Rightarrow 4 = 4m + \frac{1}{m}$

Hence, $m = \frac{1}{2}$

$\therefore$ equation of tangent is $y = \frac{x}{2} + 1 = mx + c$

Hence, c = 1

Now, $y = \frac{x}{2} + 1$ is tangent to the circle $x^2 + y^2 = 5r^2$.

$\Rightarrow \left|\frac{2}{\sqrt{5}}\right| = \sqrt{5}\,r \Rightarrow r = \frac{2}{5}$

$\therefore$ $25r^2 + \lambda + 2m + c = 4 + 4 + 1 + 1 = 10$. **Ans.**]

30. 6

Sol. Let the equation parabola be $(y-\alpha)^2 = 4\beta(x-\gamma)$

Equation of tangent is $y - \alpha = \frac{-a}{b}(x-\gamma) - \frac{\beta b}{a}$

Comparing with ax + by = 1, we get

$\gamma a^2 - \beta b^2 + \alpha ab - a = 0$

$\Rightarrow \frac{\gamma}{2} = \frac{\beta}{3} = \frac{\alpha}{4} = 1$

Hence, equation of parabola is

$(y-4)^2 = 12(x-2)$]