JEE MAIN : CHAPTER WISE TEST PAPER-10

SUBJECT :- MATHEMATICS DATE..
CLASS :- 11th NAME..
CHAPTER :- CONIC SECTION SECTION..

(SECTION-A)

1. The length of the normal (terminated by the major axis) at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(A) $\frac{b}{a}(r+r_1)$

(B) $\frac{b}{a}|r-r_1|$

(C) $\frac{b}{a}\sqrt{r\,r_1}$

(D) independent of r, r_1 where r and r_1 are the focal distances of the point.

2. The eccentric angle of the point where the line, $5x-3y=8\sqrt{2}$ is a normal to the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\tan^{-1}2$

3. The tangent at a point whose eccentric angle 60° on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) meet the auxiliary circle at L and M. If LM subtends a right angle at the centre, then eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{7}}$ (B) $\frac{2}{\sqrt{7}}$ (C) $\frac{3}{\sqrt{7}}$ (D) $\frac{1}{2}$

4. Tangent drawn to an ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ at the point with eccentric angle 30° cuts the director circle of ellipse at P and Q. The area of triangle OPQ is (where O is origin)

(A) $\frac{4\sqrt{19}}{7}$ (B) $\frac{2\sqrt{19}}{7}$

(C) $\frac{6\sqrt{19}}{7}$ (D) $\frac{8\sqrt{19}}{7}$

5. Tangent to the ellipse $\frac{x^2}{32}+\frac{y^2}{18}=1$ having slope $\frac{-3}{4}$ meet the coordinate axes in A and B. The area of ΔAOB (O is origin) equals

(A) 12 (B) 8 (C) 24 (D) 32

6. The equation of an ellipse whose focus is (–1, 1), eccentricity is 1/2 and the directrix is x – y + 3 = 0 is.

(A) $7x^2+7y^2+2xy+10x-10y+7=0$

(B) $7x^2+7y^2+2xy-10x-10y+7=0$

(C) $7x^2+7y^2+2xy-10x+10y+7=0$

(D) None of these

7. The foci of an ellipse are (±2, 0) and its eccentricity is 1/2, the equation of ellipse is.

(A) $\frac{x^2}{16}+\frac{y^2}{9}=1$ (B) $\frac{x^2}{16}+\frac{y^2}{12}=1$

(C) $\frac{x^2}{4}+\frac{y^2}{2}=1$ (D) None of these

8. Let S ≡ (3, 4) and S' ≡ (9, 12) be two foci of an ellipse. If the coordinates of the foot of the perpendicular from focus S to a tangent of the ellipse is (1, – 4) then the eccentricity of the ellipse is

(A) $\frac{4}{5}$ (B) $\frac{5}{7}$ (C) $\frac{5}{13}$ (D) $\frac{7}{13}$

9. If the circle $x^2+y^2=k^2$ and the rectangular hypererbola xy = k have no points in common then the number of integral values of k, is

(A) 0 (B) 1 (C) 2 (D) 3

10. If the chord of the hypererbola $x^2-y^2=9$ touches the parabola $y^2=12x$, then the locus of the middle points of these chord is

(A) $x^3=(x-3)y^2$ (B) $x^3=(x+3)y^2$

(C) $x(x^2-y^2)=3y$ (D) $x^3=x-3y^2$

11. A tangent to the hypererbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ meets ellipse $x^2+4y^2=4$ at two distinct points. The locus of mid point of this chord is

(A) $(x^2+4y^2)^2=4(x^2-4y^2)$

(B) $(x^2-4y^2)=4(x^2+4y^2)$

(C) $(x^2-4y^2)^2=4(4x^2+y^2)$

(D) $(x^2+4y^2)^2=4(4x^2-y^2)$

12. Consider the hyperbola H given by the equation xy = 6 and the point P (3, 2) lies on H. Let L be the line tangent to H at P (3, 2). If L intersects the positive x-axis at a point A and the positive y-axis at a point B, then the area of the triangle ΔAOB, is

(A) 12 (B) 6 (C) 10 (D) 18

13. If the curves $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $x^2 = cy$ touch each other at the point $\left(2\sqrt{2},\,4\right)$ then the value of $(a^2 + b^2 + c)$ is equal to
(A) 38 (B) 26 (C) 22 (D) 18

14. If a hyperbola whose foci are (–2, 4) and (4, 6) touches y-axis, then equation of hyperbola is

(A) $\frac{(3x+y-8)^2}{2}-\frac{(x-3y+14)^2}{8}=1$

(B) $\frac{(x-3y+14)^2}{2}-\frac{(3x+y-8)^2}{8}=20$

(C) $\frac{(x+3y-7)^2}{2}-\frac{(x-3y+8)^2}{8}=1$

(D) $\frac{(3x+y-8)^2}{1}-\frac{(x-3y+14)^2}{4}=20$

15. The parabola $y = x^2 - 9$ and $y = kx^2$ intersect each other at the points A and B. If the length AB is equal to 10 units then the value of k is equal to

(A) 75 (B) $\frac{9}{25}$ (C) $\frac{16}{25}$ (D) $\frac{16}{9}$

16. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis in P_1 and P_2. If S is the focus of the parabola then $\frac{1}{l(SP_1)}+\frac{1}{l(SP_2)}$ is equal to

(A) $\frac{4}{a}$ (B) $\frac{2}{a}$ (C) $\frac{1}{a}$ (D) $\frac{1}{4a}$

17. Let S be the focus of $y^2 = 4x$ and a point P is moving on the curve such that it's abscissa is increasing at the rate of 4 units/sec, then the rate of increase of projection of SP on $x + y = 1$ when P is at (4, 4) is

(A) $\sqrt{2}$ (B) – 1

(C) $-\sqrt{2}$ (D) $-\frac{3}{\sqrt{2}}$

18. From the point (4 , 6) a pair of tangent lines are drawn to the parabola, $y^2 = 8x$. The area of the triangle formed by these pair of tangent lines & the chord of contact of the point (4 , 6) is :

(A) 8 (B) 4
(C) 2 (D) 6

19. A tangent is drawn to the parabola $y^2 = 4x$ at the point 'P' whose abscissa lies in the interval [1,4]. The maximum possible area of the triangle formed by the tangent at 'P' , ordinate of the point 'P' and the x-axis is equal to

(A) 8 (B) 16
(C) 24 (D) 32

20. Point A is the vertex of the parabola whose equation is $y = x^2 - 2$. Points B and C are the intersections of the parabola with the circle whose equation is $x^2 + y^2 = 8$. The number of quare units in the area of ΔABC, is

(A) 8 (B) 4
(C) 12 (D) 16

(SECTION-B)

21. Let A and B be two points on the major axis of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, which are equidistant from the centre. If C and D are the images of these points in the line mirror $y = mx$, $m \neq 0$ then find the maximum area of quadrilateral ACBD.

22. Given the equation of the ellipse $\frac{(x-3)^2}{16}+\frac{(y+4)^2}{49}=1$, a parabola is such that its vertex is the lowest point of the ellipse and it passes through the ends of the minor axis of the ellipse. The equation of the parabola is in the form $16y = a(x - h)^2 - k$. Determine the value of $(a + h + k)$.

23. On an ellipse $\frac{x^2}{64}+\frac{y^2}{9}=1$, tangents drawn at $P_1, P_2, P_3, \ldots\ldots, P_n$ intersects the major axis at $T_1, T_2, T_3, \ldots\ldots, T_n$ respectively. If the value of $\sum_{i=1}^{n}\frac{\text{area}(\Delta P_iT_iS)\cdot\text{area}(\Delta P_iT_iS')}{(P_iT_i)^2}=18$, where S and S' are foci of ellipse, then find n.

24. Suppose an ellipse and a hyperbola have the same pair of foci on the x-axis with centres at the origin and they intersect at M (2, 2). If the eccentricity of ellipse is $\frac{1}{2}$ and eccentricity of hyperbola is $\sqrt{\frac{m}{n}}$ where m, n are coprime, then find the value of (m + n).

25. Let A_1 and A_2 are the vertices of the conic $C_1 : 4(x-3)^2 + 9(y-2)^2 - 36 = 0$ and a point P is moving in the plane such that $|PA_1 - PA_2| = 3\sqrt{2}$ then locus of P is another conic C_2. If
D_1 denotes distance between foci of the conic C_2.
D_2 denotes product of the perpendiculars from the points A_1, A_2 upon any tangent drawn to conic C_2.
and D_3 denotes length of the tangent drawn from any point on auxiliary circle of conic C_1 to the auxiliary circle of the conic C_2, then find the value of $\left(\frac{D_1 D_2}{{D_3}^2}\right)$.

26. The tangent at point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point (0, –b) and the normal at point P passes through the point $(2\sqrt{2}\ a, 0)$. If e denote the eccentricity of hyperbola then find the value of e^2.

27. Let $C_1 : |z-2| = \left|\frac{z+\bar{z}+4}{2}\right|$ and $C_2 : \left||z+\sqrt{5}|-|z-\sqrt{5}|\right| = 2$ be two curves. If from a point on the curve C_2 two mutually perpendicular tangents are drawn to the curve C_1 and area of the triangle formed by pair of tangents and their corresponding chord of contact is $k\sqrt{7}$, then find k.

28. If angle between two focal chords of a parabola $(y-5)^2 = 8(x-1)$ which are tangents to the circle $x^2 + y^2 = 9$ is $\tan^{-1}\left(\frac{a}{b}\right)$, where a and b relatively prime number then find the value of a + b.

29. If the line y = mx + c is tangent to the circle $x^2 + y^2 = 5r^2$ and the parabola $y^2 - 4x - 2y + 4\lambda + 1 = 0$ and point of contact of the tangent with the parabola is (8, 5), then find the value of $(25r^2 + \lambda + 2m + c)$.

30. If $2a^2 - 3b^2 + 4ab - a = 0$ and a variable line ax + by = 1 always touches a parabola whose axis is parallel to x-axis then the equation of parabola is $(y-p)^2 = q(x-r)$. Find the value of $(q-(p+r))$.