DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-VECTOR ALGEBRA

1 **(a)**

We have,

$|\vec{a}| = 1, |\vec{b}| = 1$ and $\vec{a}.\vec{b} = \cos\theta$

Now, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 1 + 1 - 2|\vec{a}|\,|\vec{b}|\cos\theta$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 4\sin^2\frac{\theta}{2}$

$\Rightarrow \left|\frac{\vec{a} - \vec{b}}{2}\right|^2 = \sin^2\frac{\theta}{2} \Rightarrow \left|\frac{\vec{a} - \vec{b}}{2}\right| = \sin\frac{\theta}{2}$

2 **(c)**

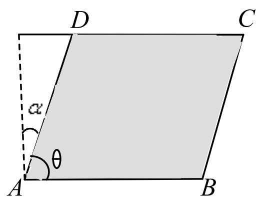
If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ are two non-zero non-collinear vectors and x, y are two scalars such that $x\vec{\mathbf{a}} + y\vec{\mathbf{b}} = 0$, then $x = 0, y = 0$.

Because otherwise one will be a scalar multiple of the other and hence collinear, which is a contradiction

3 **(b)**

$\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$



$\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{AD}} = -2 + 20 + 22 = 40$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{4 + 100 + 120} = \sqrt{225} = 15$

$|\overrightarrow{\mathbf{AD}}| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore \cos\theta = \frac{40}{45} = \frac{8}{9}$

$\therefore \theta + \alpha = 90°$

$\Longrightarrow \alpha = 90° - \theta$

$\Longrightarrow \cos\alpha = \sin\theta = \sqrt{1 - \frac{64}{81}} = \frac{\sqrt{17}}{9}$

4 **(a)**

Let $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Sience, $\frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|} = \frac{1}{\sqrt{30}}$

$\Rightarrow \frac{(x\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}})}{|\sqrt{4+1+25}|} = \frac{1}{\sqrt{30}}$

$\Rightarrow 2x - 1 + 5 = 1$

$\Rightarrow x = -\frac{3}{2}$

5 **(b)**

Now, $2\vec{\mathbf{a}} - \vec{\mathbf{c}} = 2(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$= \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\vec{\mathbf{a}} + \vec{\mathbf{b}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$let θ be the angle between $2\vec{\mathbf{a}} - \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$.

$\therefore \cos\theta = \frac{(\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{1^2+1^2}\sqrt{1^2+1^2}}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

6 **(d)**

Since $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ are collinear with $\vec{c}$ and $\vec{a}$ respectively. Therefore, there exist scalars x,y such that $\vec{a} + \vec{b} = x\vec{c}$ and $\vec{b} + \vec{c} = y\vec{a}$. Now,

$\vec{a} + \vec{b} = x\vec{c} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (x+1)\vec{c}$ ...(i)

and,

$\vec{b} + c = y\vec{a} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (y+1)\vec{a}$ ....(ii)

From (i) and (ii), we get

$(x+1)\vec{c} = (y+1)\vec{a}$

If $x \neq -1$, then

$(x+1)\vec{c} = (y+1)\vec{a} \Rightarrow \vec{c} = \frac{y+1}{x+1}\vec{a}$

$\Rightarrow \vec{c}$ and $\vec{a}$ are collinear

This is a contradiction to the given condition. Therefore, $x = -1$

Putting $x = -1$ in $\vec{a} + \vec{b} = x\vec{c}$, we get

$\vec{a} + \vec{b} + \vec{c} = (-1+1)\vec{c} = \vec{0}$

7 **(b)**

We have, $[\vec{\mathbf{a}}\ \vec{\mathbf{b}} + \vec{\mathbf{c}}\ \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}]$

$= \vec{\mathbf{a}} \cdot [(\vec{\mathbf{b}} + \vec{\mathbf{c}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})]$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}} \times \vec{\mathbf{c}})$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} - \vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}}) + \vec{\mathbf{a}} \cdot (\vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] + [\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}] = 0$

8 **(a)**

It is given that points P, Q and R with position vectors $60\hat{i} + 3\hat{j}$, $40\hat{i} - 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively are collinear

$\therefore \vec{PQ} = \lambda\, \vec{QR}$ for some scalar λ

$\Rightarrow -20\hat{i} - 11\hat{j} = \lambda\{(a - 40)\hat{i} - 44\hat{j}\}$

$\Rightarrow \lambda(a - 40) = -20,\ -11 = -44\,\lambda$

$\Rightarrow \lambda = \frac{1}{4}$ and $a = -40$

9 **(a)**

Required unit vector

$$\vec{\mathbf{c}} = \frac{\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})}{|\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})|}$$

Now,

$\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}$

$= 3(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 6(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= -9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$$\therefore\ \vec{\mathbf{c}} = \frac{-9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}}{\sqrt{9^2 + 9^2}} = \pm \frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

10 **(b)**

$$\begin{vmatrix} 2 & 1 & 4 \\ 4 & -2 & 3 \\ 2 & -3 & -\lambda \end{vmatrix} = 0$$

$\Longrightarrow 2(2\lambda + 9) - 1(-4\lambda - 6) + 4(-12 + 4) = 0$

$\Longrightarrow 4\lambda + 18 + 4\,\lambda + 6 - 48 + 16 = 0$

$\Longrightarrow 8\lambda = 8$

$\Longrightarrow \lambda = 1$

11 **(b)**

We have,

$$[\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} al + a_1l_1 & am + a_1m_1 & an + a_1n_1 \\ bl + b_1l_1 & bm + b_1m_1 & bn + b_1n_1 \\ cl + c_1l_1 & cm + c_1m_1 & cn + a_1n_1 \end{vmatrix}$$

$$\Rightarrow [\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} a & a_1 & 0 \\ b & b_1 & 0 \\ c & c_1 & 0 \end{vmatrix} \begin{vmatrix} l & l_1 & 0 \\ m & m_1 & 0 \\ n & n_1 & 0 \end{vmatrix} = 0$$

Hence, the given vectors are coplanar

13 **(a)**

Given that $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\therefore \vec{a} \perp \vec{b} \times \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) = 0 \Rightarrow [\vec{a}\vec{b}\vec{c}] = 0$

14 **(c)**

$(\vec{\mathbf{d}} + \vec{\mathbf{a}}) \cdot [\vec{\mathbf{a}} \times \{\vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{d}})\}]$

$= (\vec{\mathbf{d}} + \vec{\mathbf{a}}) \cdot [\vec{\mathbf{a}} \times \{(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})\vec{\mathbf{c}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})\vec{\mathbf{d}}\}]$

$= (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{d}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{c}})] - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})[\vec{\mathbf{d}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{d}})]$

$+ (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{a}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{c}})] - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})[\vec{\mathbf{a}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{d}})]$

$= (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{d}}\, \vec{\mathbf{a}}\, \vec{\mathbf{c}}] = (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{a}}\, \vec{\mathbf{c}}\, \vec{\mathbf{d}}]$

16 **(a)**

Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \vec{\mathbf{b}} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = \lambda\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore [\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}] = 0$

$$\Longrightarrow \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ \lambda & -1 & 2 \end{vmatrix} = 0$$

$\Longrightarrow 1(6 - 4) + 2(-4 + 4\lambda) + 3(2 - 3\lambda) = 0$

$\Longrightarrow \lambda = 0$

17 **(b)**

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

$|\vec{\mathbf{a}}|^2 = a_1^2 + a_2^2 + a_3^2$

and $\vec{\mathbf{a}} \times \hat{\mathbf{i}} = (a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}) \times \hat{\mathbf{i}}$

$= -a_2\hat{\mathbf{k}} + a_3\hat{\mathbf{j}}$

$(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 = a_2^2 + a_3^2$

Similarly, $(\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 = a_3^2 + a_1^2$

and $(\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2 = a_1^2 + a_2^2$

Now, $(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2$

$= a_2^2 + a_3^2 + a_3^2 + a_1^2 + a_1^2 + a_2^2$

$= 2(a_1^2 + a_2^2 + a_3^2) = 2(\vec{\mathbf{a}})^2$

18 **(d)**

Since, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = 2\hat{\mathbf{i}} - 4\hat{\mathbf{k}}, \vec{\mathbf{c}} = \hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ are coplanar.

$$\therefore [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & -4 \\ 1 & \lambda & 3 \end{vmatrix} = 0$$

$\Rightarrow 4\lambda - 1(6 + 4) + 2\lambda = 0$

$\Rightarrow 6\lambda = 10 \Rightarrow \lambda = \frac{5}{3}$

20 **(c)**

$\vec{\mathbf{A}}, \vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ are three vectors, then volume of parallelepiped

$V = [\, \vec{\mathbf{A}} \;\; \vec{\mathbf{B}} \;\; \vec{\mathbf{C}} \,]$

$$= \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 + a^3 - a$$

$\Rightarrow V = 1 + a^3 - a$

On differentiating with respect to a, we get

$$\frac{dV}{da} = 3a^2 - 1 = 0$$

For maximum or minimum, put $\frac{dV}{a} = 0$

$$\Rightarrow a = \pm \frac{1}{\sqrt{3}}$$

$\frac{d^2V}{da^2} = 6a$, positive at $a = \frac{1}{\sqrt{3}}$.

$\therefore V$ is minimum at $a = \frac{1}{\sqrt{3}}$.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	A	B	D	B	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	A	C	B	A	B	D	C	C