DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-VECTOR ALGEBRA

1 **(a)**

The point of intersection of $\vec{\mathbf{r}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$ and $\vec{\mathbf{r}} \times \vec{\mathbf{b}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}}$ is $\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

$\therefore\ \vec{\mathbf{r}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) + (2\hat{\mathbf{i}} - \hat{\mathbf{k}}) = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

2 **(a)**

Since $\vec{a}, \vec{b}$ and $a \times \vec{b}$ are non-coplanar vectors

$\therefore \vec{r} = x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})$ for some scalars x, y, z ...(i)

Now,

$\vec{b} = \vec{r} \times \vec{a}$

$\Rightarrow \vec{b} = \{x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})\} \times \vec{a}$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) + z((\vec{a} \times \vec{b}) \times \vec{a})$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) - z(\vec{a} \times (\vec{a} \times \vec{b}))$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) - z\{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\}$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) + z(\vec{a} \cdot \vec{a})\vec{b}\quad [\because \vec{a} \cdot \vec{b} = 0]$

Comparing the coefficients, we get

$$y = 0, z = \frac{1}{\vec{a} \cdot \vec{a}} = \frac{1}{|\vec{a}|^2}$$

Putting the values of y and z in (i), we get

$$\vec{r} = x\vec{a} + \frac{1}{|\vec{a}|^2}(\vec{a} \times \vec{b})$$

4 **(b)**

$(\vec{\mathbf{u}} + \vec{\mathbf{v}} - \vec{\mathbf{w}}).[(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \times (\vec{\mathbf{v}} - \vec{\mathbf{w}})]$

$= (\vec{\mathbf{u}} + \vec{\mathbf{v}} - \vec{\mathbf{w}}).[\vec{\mathbf{u}} \times \vec{\mathbf{v}} - \vec{\mathbf{u}} \times \vec{\mathbf{w}} + \vec{\mathbf{v}} \times \vec{\mathbf{w}}]$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} - \vec{\mathbf{v}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{w}} - \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}} - \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}}$

5 **(d)**

$\therefore\ \vec{\mathbf{p}} - 2\,\vec{\mathbf{q}} = 7\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} - 2(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$= \hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{p}} - 2\vec{\mathbf{q}}| = \sqrt{1^2 + (-4)^2 + (-7)^2} = \sqrt{66}$

7 **(a)**

$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= -\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$\Rightarrow (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\hat{\mathbf{k}}$

Now, $\lambda \vec{\mathbf{a}} + \mu \vec{\mathbf{b}} = \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$= (\lambda + \mu)\hat{\mathbf{i}} + (\lambda + \mu)\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$

$\because \lambda \vec{\mathbf{a}} + \mu \vec{\mathbf{b}} = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$

$\Rightarrow (\lambda + \mu)\hat{\mathbf{i}} + (\lambda + \mu)\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}} = -\hat{\mathbf{k}}$

On equating the coefficient of $\hat{\mathbf{i}}$ we get $\lambda + \mu = 0$

13 **(d)**

We have,

$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$

$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$ or, $\vec{b} - \vec{c} = 0 \Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$ or, $\vec{b} = \vec{c}$

14 **(c)**

Given that, $|\vec{\mathbf{a}}| = 2\sqrt{2}, |\vec{\mathbf{b}}| = 3$

The longer vectors is $5\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + \vec{\mathbf{a}} - 3\vec{\mathbf{b}} = 6\vec{\mathbf{a}} - \vec{\mathbf{b}}$

Length of one diagonal

$= |6\vec{\mathbf{a}} - \vec{\mathbf{b}}|$

$= \sqrt{36\vec{\mathbf{a}}^2 + \vec{\mathbf{b}}^2 - 2 \times 6|\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}| \cos 45^\circ}$

$= \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}}$

$= \sqrt{288 + 9 - 12 \times 6} = \sqrt{225} = 15$

Other diagonal is $4\vec{\mathbf{a}} + 5\vec{\mathbf{b}}$.

Its length $= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

15 **(a)**

Given projection of $\vec{\mathbf{a}}$ on $\vec{\mathbf{b}} = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$\Rightarrow \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|} = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$\Rightarrow \frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta}{|\vec{\mathbf{b}}|} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$

$\Rightarrow \tan\theta = \frac{1}{|\vec{\mathbf{b}}|}$

$\Rightarrow \tan\theta = \frac{1}{\frac{1}{3}\sqrt{1^2 + 1^2 + 1^2}}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

17 **(c)**

Since, $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} = k\vec{\mathbf{c}}$

$\therefore\ \vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 6\vec{\mathbf{c}} = k\vec{\mathbf{c}} + 6\vec{\mathbf{c}}$

$= (k+6)\vec{\mathbf{c}} = \lambda\vec{\mathbf{c}}\ \ (\because \lambda \neq 0)$

18 **(d)**

$$\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = -2\hat{\mathbf{k}}$$

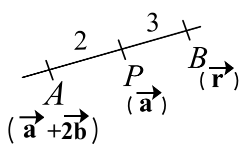
$$\therefore |\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}| = \frac{|\vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}|}{|\vec{\mathbf{u}} \times \vec{\mathbf{v}}|}$$

$$\Rightarrow |\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}| = \frac{|-6\hat{\mathbf{k}}|}{|-2\hat{\mathbf{k}}|} = 3$$

19 **(c)**

Let the position of B is $\vec{\mathbf{r}}$.

$$\therefore\ \vec{\mathbf{a}} = \frac{2\vec{\mathbf{r}} + 3(\vec{\mathbf{a}} + 2\vec{\mathbf{b}})}{2+3}$$



$\Rightarrow 5\vec{\mathbf{a}} = 2\vec{\mathbf{r}} + 3\vec{\mathbf{a}} + 6\vec{\mathbf{b}}$

$\Rightarrow 2\vec{\mathbf{r}} = 2\vec{\mathbf{a}} - 6\vec{\mathbf{b}}$

$\therefore\ \vec{\mathbf{r}} = \vec{\mathbf{a}} - 3\vec{\mathbf{b}}$

20 **(a)**

Since, $(\vec{\mathbf{A}} + t\vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = 0$ [given]

$\Rightarrow \left[(1-t)\hat{\mathbf{i}} + (2+2t)\hat{\mathbf{j}} + (3+t)\hat{\mathbf{k}}\right] \cdot \left(3\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) = 0$

$\Rightarrow 3(1-t) + (2+2t) = 0 \Rightarrow t = 5$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	B	B	D	A	A	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	D	C	A	C	C	D	C	A