DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-VECTOR ALGEBRA

1 **(d)**

$$\begin{vmatrix} 7 & -11 & 1 \\ 5 & 3 & -2 \\ 12 & -8 & -1 \end{vmatrix}$$

$= 7(-3-16) + 11(-5+24) + 1(-40-36)$

$= -133 + 209 - 76 = 0$

$\therefore$ Vector are collinear.

2 **(c)**

Let the position vectors of the points A, B, C are $\vec{\mathbf{0}}, \vec{\mathbf{a}} + \vec{\mathbf{b}}, \vec{\mathbf{a}} - \vec{\mathbf{b}}$ respectively and $\theta = 90°$

$\therefore$ Area of triangle $= \frac{1}{2}|\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}|$

$= \frac{1}{2}|(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} - \vec{\mathbf{b}})|$

$= \frac{1}{2}|2\vec{\mathbf{b}} \times \vec{\mathbf{a}}|$

$= |\vec{\mathbf{b}}|\ |\vec{\mathbf{a}}| \sin\theta = 3 \times 2 \sin 90° = 6$

3 **(a)**

We have, $|[\vec{a}\ \vec{b}\ \vec{c}]| = V$

Let V_1 be the volume of the parallelopiped formed by the vectors $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$. Then,

$V_1 = |[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]|$

Now,

$$[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$$

$\Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = [\vec{a}\ \vec{b}\ \vec{c}]^2 [\vec{a}\vec{b}\ \vec{c}]$

$\Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = [\vec{a}\ \vec{b}\ \vec{c}]^3$

$\therefore V_1 = |[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]| = |[\vec{a}\ \vec{b}\ \vec{c}]^3| = V^3$

4 **(a)**

Let l, m, n be the direction cosines of the required vector. As it makes equal angles with X and Y axes

$\therefore l = m$

$\therefore$ Required vector $\vec{r} = l\hat{i} + m\hat{j} + n\hat{k} = l\hat{i} + l\hat{j} + n\hat{k}$

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow 2l^2 + n^2 = 1$ ...(i)

Since, $\vec{r}$ is perpendicular to $-\hat{i}+2\hat{j}+2\hat{k}$

$\therefore \vec{r}\cdot(-\hat{i}+2\hat{j}+2\hat{k})=0 \Rightarrow -l+2l+2n=0 \Rightarrow l+2n=0$ ....(ii)

From (i) and (ii), we get $n \mp \frac{1}{3}, l = \mp \frac{2}{3}$

Hence, $\vec{r}=\frac{1}{3}(\pm 2\hat{i} \pm 2\hat{j} \mp \hat{k}) = \pm \frac{1}{3}(2\hat{i}+2\hat{j}-\hat{k})$

5 **(a)**

Let the required vector be $\vec{a}$. Then, $\hat{i}-\hat{j}, \hat{i}+\hat{j}$ and $\vec{a}$ form a right handed system

$\therefore \vec{a}=(\hat{i}-\hat{j})\times(\hat{i}+\hat{j})=\hat{k}+\hat{k}=2\hat{k}$

Hence, the required unit vector $\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\hat{k}$

6 **(b)**

$\vec{\mathbf{p}}=x\vec{\mathbf{a}}+y\vec{\mathbf{b}}+z\vec{\mathbf{c}}$

$\Rightarrow 3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}=x(\hat{\mathbf{i}}+\hat{\mathbf{j}})+y(\hat{\mathbf{j}}+\hat{\mathbf{k}})+z(\hat{\mathbf{i}}+\hat{\mathbf{k}})$

$\Rightarrow 3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}=(x+z)\hat{\mathbf{i}}+(x+y)\hat{\mathbf{j}}+(y+z)\hat{\mathbf{k}}$

On comparing both sides the coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, we get

$x+z=3$ ....(i)

$x+y=2$....(ii)

and $y+z=4$ (iii)

on solving Eqs. (i), (ii) and (iii), we get

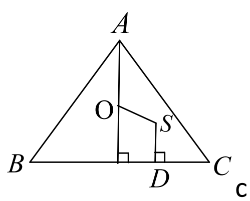
$x=\frac{1}{2}, y=\frac{3}{2}, z=\frac{5}{2}$

7 **(a)**

From geometry

$\overrightarrow{\mathbf{AO}}=2\overrightarrow{\mathbf{SD}}$

Where D is the mind point of BC



$\therefore \overrightarrow{\mathbf{SA}}+\overrightarrow{\mathbf{SB}}+\overrightarrow{\mathbf{SC}}$

$=\overrightarrow{\mathbf{SA}}+2\overrightarrow{\mathbf{SD}}$ $(\because \overrightarrow{\mathbf{SB}}+\overrightarrow{\mathbf{SC}}=2\overrightarrow{\mathbf{SD}})$

$=\overrightarrow{\mathbf{SA}}+\overrightarrow{\mathbf{AO}}$

$=\overrightarrow{\mathbf{SO}}$

8 **(c)**

We have,

$\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta=0$ and $|\vec{a}||\vec{b}|\sin\theta=0$

$\Rightarrow (|\vec{a}|=0$ or, $|\vec{b}|=0$ or, $\cos\theta=0)$

And,

$(|\vec{a}|=0$ or, $|\vec{b}|=0$ or, $\sin\theta=0)$

$\Rightarrow|\vec{a}| = 0$ or, $|\vec{b}| = 0$ $\left[\begin{array}{c}\because \cos\theta \text{ and } \sin\theta \\ \text{are not zero zimultaneously}\end{array}\right]$

10 **(c)**

Given $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2$

$\Longrightarrow 4\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0 \Longrightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

So, angle between them is 90°

11 **(c)**

We have,

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$

$\Rightarrow (\vec{r} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \vec{r} - \vec{b}$ is parallel to $\vec{a}$

$\Rightarrow \vec{r} - \vec{b} = \lambda\vec{a}$ for some scalar λ

$\Rightarrow \vec{r} - \vec{b} + \lambda\vec{a}$ ....(i)

Now,

$\vec{r} \perp \vec{c}$

$\Rightarrow \vec{r} \cdot \vec{c} \cdot \vec{c} = 0$

$\Rightarrow \vec{b} \cdot \vec{c} + \lambda(\vec{a} \cdot \vec{c}) = 0 \Rightarrow \lambda = -\dfrac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{c}}$

Putting the value of λ in (i), we get

$\vec{r} = \vec{b} - \left(\dfrac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{c}}\right)\vec{a}$

12 **(d)**

We have, $|\vec{\alpha}| = 1 = |\vec{\beta}|$ and $\vec{\alpha} \cdot \vec{\beta} = 0$

Now,

$\vec{\gamma} = x\vec{\alpha} + y\vec{\beta} + z(\vec{\alpha} \times \vec{\beta})$

$\Rightarrow \vec{\alpha} \cdot \vec{\gamma} = x(\vec{\alpha} \cdot \vec{\alpha}) + y(\vec{\alpha} \cdot \vec{\beta}) + z\{\vec{\alpha} \cdot (\vec{\alpha} \times \vec{\beta})\}$

$\vec{\beta} \cdot \vec{\gamma} = x(\vec{\beta} \cdot \vec{\alpha}) + y(\vec{\beta} \cdot \vec{\beta}) + z\{\vec{\beta} \cdot (\vec{\alpha} \times \vec{\beta})\}$

And,

$(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\gamma} = x\{\vec{\alpha} \cdot (\vec{\alpha} \times \vec{\beta}) + y\{\vec{\beta} \cdot (\vec{\alpha} \times \vec{\beta})\} + z\{(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\beta})\}$

$\Rightarrow \cos\theta = x, \cos\theta = y$ and $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = z|\vec{\alpha} \times \vec{\beta}|^2$

$\Rightarrow x = \cos\theta, y = \cos\theta$ and $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = z$

$[\because |\vec{\alpha} \times \vec{\beta}| = |\vec{\alpha}|\,|\vec{\beta}| \sin 90° = 1]$

Now,

$$[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]^2 = \begin{vmatrix} \vec{\alpha} \cdot \vec{\alpha} & \vec{\alpha} \cdot \vec{\beta} & \vec{\alpha} \cdot \vec{\gamma} \\ \vec{\beta} \cdot \vec{\alpha} & \vec{\beta} \cdot \vec{\beta} & \vec{\beta} \cdot \vec{\gamma} \\ \vec{\gamma} \cdot \vec{\alpha} & \vec{\gamma} \cdot \vec{\beta} & \vec{\gamma} \cdot \vec{\gamma} \end{vmatrix}$$

$$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]^2 = \begin{vmatrix} 1 & 0 & \cos\theta \\ 0 & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix} = 1 - 2\cos^2\theta$$

$\Rightarrow z^2 = 1 - 2x^2$

Also, $z^2 = 1 - 2y^2$ and $z^2 = 1 - x^2 - y^2$

13 **(a)**

$(\vec{\mathbf{a}}-\vec{\mathbf{d}})\times(\vec{\mathbf{b}}-\vec{\mathbf{c}})$

$=\vec{\mathbf{a}}\times\vec{\mathbf{b}}-\vec{\mathbf{a}}\times\vec{\mathbf{c}}-\vec{\mathbf{d}}\times\vec{\mathbf{b}}+\vec{\mathbf{d}}\times\vec{\mathbf{c}}$

$=\vec{\mathbf{c}}\times\vec{\mathbf{d}}-\vec{\mathbf{b}}\times\vec{\mathbf{d}}-\vec{\mathbf{d}}\times\vec{\mathbf{b}}+\vec{\mathbf{d}}\times\vec{\mathbf{c}}=\vec{\mathbf{0}}$

$[\because\ \vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{c}}\times\vec{\mathbf{d}}, \vec{\mathbf{a}}\times\vec{\mathbf{c}}=\vec{\mathbf{b}}\times\vec{\mathbf{d}}$, given]

$\Longrightarrow(\vec{\mathbf{a}}-\vec{\mathbf{d}})||(\vec{\mathbf{b}}-\vec{\mathbf{c}})$

$\Longrightarrow\vec{\mathbf{a}}-\vec{\mathbf{d}}=\lambda(\vec{\mathbf{b}}-\vec{\mathbf{c}})$

14 **(a)**

Since $\vec{a},\vec{b},\vec{c}$ are non-coplanar unit vectors

$\therefore[\vec{a}\vec{b}\vec{c}]=$ Volume of a parallelopiped whose each edge is of one unit length

$\Rightarrow[\vec{a}\vec{b}\vec{c}]=\pm 1$

16 **(d)** Let D be the mid-point of BC. Then,

$\vec{AB}+\vec{AC}=2\vec{AD}$

$\Rightarrow 2\,\vec{AD}=8\hat{i}+2\hat{j}+8\hat{k}$

$\Rightarrow\vec{AD}=4\hat{i}+\hat{j}+4\hat{k}$

$\Rightarrow|\vec{AD}|=\sqrt{16+1+16}=\sqrt{33}$

17 **(c)**

$\therefore$ Median vector through $\vec{\mathbf{A}}=\frac{1}{2}(\overrightarrow{\mathbf{AB}}+\overrightarrow{\mathbf{AC}})$

$=\frac{1}{2}\left[\left(3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)+\left(5\hat{\mathbf{i}}-5\hat{\mathbf{j}}+2\hat{\mathbf{k}}\right)\right]$

$=4\hat{\mathbf{i}}+3\hat{\mathbf{k}}$

$\therefore$ Length of the median $=\sqrt{4^2+3^2}=5$ units

18 **(d)**

Given, $(\vec{\mathbf{a}}-\lambda\,\vec{\mathbf{b}})\cdot(\vec{\mathbf{b}}-2\vec{\mathbf{c}})\times(\vec{\mathbf{c}}+2\vec{\mathbf{a}})=0$

$\Longrightarrow(\vec{\mathbf{a}}-\lambda\,\vec{\mathbf{b}})\cdot\{\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{b}}\times 2\vec{\mathbf{a}}-4(\vec{\mathbf{c}}\times\vec{\mathbf{a}})\}=0$

$\Longrightarrow\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+\vec{\mathbf{a}}.(\vec{\mathbf{b}}\times 2\vec{\mathbf{a}})-\vec{\mathbf{a}}.4(\vec{\mathbf{c}}\times\vec{\mathbf{a}})$

$-\lambda\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\lambda\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times 2\vec{\mathbf{a}})+4\lambda\,\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})=0$

$\Longrightarrow\vec{\mathbf{a}}(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+4\lambda\vec{\mathbf{b}}.(\vec{\mathbf{c}}\times\vec{\mathbf{a}})=0$

$\Longrightarrow\{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\}(1+4\,\lambda)=0$

$\Longrightarrow\lambda=-\frac{1}{4}\ \ [\because\ \vec{\mathbf{a}}.(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\neq 0$,given$]$

20 **(d)** $\therefore$ Total force $\vec{\mathbf{P}}=\vec{\mathbf{P}}_1+\vec{\mathbf{P}}_2+\vec{\mathbf{P}}_3$

$=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}=2\hat{\mathbf{j}}$

and displacement $\overrightarrow{\mathbf{AB}}=6\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}-\left(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)$

$=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$\therefore$ Work done $=\vec{\mathbf{P}}\cdot\overrightarrow{\mathbf{AB}}$

$=2\hat{\mathbf{j}}\left(2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)=8$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	A	A	B	A	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	A	A	C	D	C	D	A	D