DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-VECTOR ALGEBRA

1 **(c)**

Let $\vec{\alpha} = \lambda\,\vec{\mathbf{a}} + \mu\vec{\mathbf{b}} + t\vec{\mathbf{c}}$ ...(i)

Now, $\vec{\mathbf{a}}\cdot\vec{\mathbf{p}} = \vec{\mathbf{b}}\cdot\vec{\mathbf{q}} = \vec{\mathbf{c}}\cdot\vec{\mathbf{r}} = 1$

$\Rightarrow\ \vec{\alpha}\cdot\vec{\mathbf{p}} = \lambda\,(\,\vec{\mathbf{a}}\cdot\vec{\mathbf{p}}) + 0 + 0$

$\Rightarrow \lambda = \vec{\alpha}\cdot\vec{\mathbf{p}}$

Similarly, $\mu = \vec{\alpha}\cdot\vec{\mathbf{q}}$

and $t = \vec{\alpha}\cdot\vec{\mathbf{r}}$

From Eq. (i), we get

$\vec{\alpha} = (\,\vec{\alpha}\cdot\vec{\mathbf{p}})\vec{\mathbf{a}} + (\,\vec{\alpha}\cdot\vec{\mathbf{q}}\,)\vec{\mathbf{b}} + (\vec{\alpha}\cdot\vec{\mathbf{r}})\vec{\mathbf{c}}$

2 **(a)**

Since, $\vec{\mathbf{b}}\times\vec{\mathbf{c}}$ is a vector perpendicular to $\vec{\mathbf{b}},\vec{\mathbf{c}}$. Therefore $\vec{\mathbf{a}}\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$ is a vector again in plane of $\vec{\mathbf{b}},\vec{\mathbf{c}}$.

3 **(c)**

$(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{b}} + \vec{\mathbf{b}}\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})$

$= (\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{b}} + (\vec{\mathbf{b}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{b}}$

$= \vec{\mathbf{a}}\quad [\because |\vec{\mathbf{b}}| = 1]$

4 **(d)**

$$\because \sum_{i=1}^{n}\vec{\mathbf{a}}_i = \vec{\mathbf{0}}$$

$$\therefore \left(\sum_{i=1}^{n}\vec{\mathbf{a}}_i\right)\left(\sum_{i=1}^{n}\vec{\mathbf{a}}_j\right) = \sum_{i=1}^{n}|\vec{\mathbf{a}}_i|^2 + 2\sum_{1\le i<}\ \sum_{j\le n}\vec{\mathbf{a}}_i\cdot\vec{\mathbf{a}}_j$$

$$\Rightarrow 0 = n + 2\sum_{1\le i<}\ \sum_{j\le n}\vec{\mathbf{a}}_i\cdot\vec{\mathbf{a}}_j$$

$$\therefore \sum_{1\le i<}\ \sum_{j\le n}\vec{\mathbf{a}}_i\cdot\vec{\mathbf{a}}_j = -\frac{n}{2}$$

5 **(b)**

Since, given vectors are perpendicular.

$\therefore (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}})\cdot(6\hat{\mathbf{i}} - \hat{\mathbf{j}} + c\hat{\mathbf{k}}) = 0$

$\Longrightarrow 18 + 2 - 5c = 0 \Longrightarrow c = 4$

6 **(d)**

Given, $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \vec{\mathbf{0}}$ and $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = 0$

$\Longrightarrow \vec{\mathbf{a}}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$ which is possible only if

$\vec{\mathbf{a}} = \vec{\mathbf{0}}$ or $\vec{\mathbf{b}} = \vec{\mathbf{0}}$

7 **(a)**

Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

First diagonal, $\vec{\mathbf{a}} + \vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 7$

Second diagonal, $\vec{\mathbf{a}} - \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{a}} - \vec{\mathbf{b}}| = \sqrt{69}$

8 **(b)**

Given $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\Rightarrow \vec{\mathbf{a}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{a}} \times \vec{\mathbf{c}} = 0$

$\Rightarrow \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

Similarly, $\vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

$\therefore\ \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} \neq \vec{\mathbf{0}}$

Alternate: Since, $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are unit vectors and $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{a}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$,

so $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ represent an equilateral triangle.

$\therefore\ \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} \neq \vec{\mathbf{0}}$

9 **(c)**

We have,

$\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$

$= \vec{ED} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{CD}\ \ [\because \vec{AB} = \vec{ED} \text{ and } \vec{AF} = \vec{CD}]$

$= (\vec{AC} + \vec{CD}) + (\vec{AE} + \vec{ED}) + \vec{AD}$

$= 3\vec{AD} = 6\vec{AG}\ \ [\because \vec{AD} = 2\vec{AG}]$

10 **(c)**

I. It is true that non-zero, non-collinear vectors are linearly independent.

II. It is also true that any three coplanar vectors are linearly dependent.

$\therefore$ Both I and II are true.

11 **(a)**

Let $\vec{\alpha} = 2\vec{a} - 3\vec{b}, \vec{\beta} = 7\vec{b} - 9\vec{c}$ and $\vec{\gamma} = 12\vec{c} - 23\vec{a}$

Then,

$$[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = \begin{vmatrix} 2 & -3 & 0 \\ 0 & 7 & -9 \\ -23 & 0 & 12 \end{vmatrix} [\vec{a}\,\vec{b}\,\vec{c}]$$

$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = (168 + 3 \times -207)[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = 0 \qquad [\because [\vec{a}\,\vec{b}\,\vec{c}] = 0]$

$\Rightarrow \vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are coplanar vectors

12 **(b)**

Given, $[\vec{\mathbf{a}} + \vec{\mathbf{b}}\ \vec{\mathbf{b}} + \vec{\mathbf{c}}\ \vec{\mathbf{c}} + \vec{\mathbf{a}}] = [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$

$\Rightarrow 2[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] = [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$

$= [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] = 0$

Hence, $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar.

13 **(c)**

Given, $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$ and $|\vec{\mathbf{a}}|=\sqrt{37}$, $|\vec{\mathbf{b}}|=3$, and $|\vec{\mathbf{c}}|=4$

Now, $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$

$\Rightarrow \vec{\mathbf{a}}=-(\vec{\mathbf{b}}+\vec{\mathbf{c}})$

$\Rightarrow |\vec{\mathbf{a}}|^2=\left|-(\vec{\mathbf{b}}+\vec{\mathbf{c}})\right|^2$

$\Rightarrow |\vec{\mathbf{a}}|^2=|\vec{\mathbf{b}}|^2+|\vec{\mathbf{c}}|^2+2|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\theta$

$=9+16+24\cos\theta$

$\Rightarrow 37=25+24\cos\theta$

$\Rightarrow 24\cos\theta=12 \Rightarrow \theta=60°$

14 **(a)**

Let unit vector be $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$

$\therefore$ $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$ is perpendicular to $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$,

Then $a+b+c=0$ .....(i)

Since, $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$, $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$,$(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$ are coplanar

$$\therefore \begin{vmatrix} a & b & c \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$\Longrightarrow -3a+b+c=0$ ....(ii)

From Eqs. (i) and (ii), we get

$a=0$ and $c=-b$

Also, $a^2+b^2+c^2=1$

$\Longrightarrow 0+b^2+b^2=1$

$\Longrightarrow b=\dfrac{1}{\sqrt{2}}$

$\therefore a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}=\dfrac{1}{\sqrt{2}}\hat{\mathbf{j}}-\dfrac{1}{\sqrt{2}}\hat{\mathbf{k}}$

16 **(b)**

Given, $\overrightarrow{\mathbf{OA}}=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{OB}}=5\hat{\mathbf{i}}-4\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{OC}}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

volume of parallelopiped

$=[\overrightarrow{\mathbf{OA}}\ \overrightarrow{\mathbf{OB}}\ \overrightarrow{\mathbf{OC}}]$

$$=\begin{vmatrix} 2 & -2 & 1 \\ 5 & -4 & 4 \\ 1 & -2 & 4 \end{vmatrix}$$

$=2(-16+8)+2(20-4)+1(-10+4)$

$=10$ cu units

18 **(a)**

We have,

$$\vec{a}=\lambda(\vec{b}\times\vec{c})=\lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -2 & 4 & 1 \end{vmatrix}=\lambda(-10\hat{i}-7\hat{k}+8\hat{k})$$

Now,

$\vec{a}\cdot(\hat{i}-2\hat{j}+\hat{k})=-6$

$\Rightarrow\lambda(-10+14+8)=-6\Rightarrow\lambda=-\frac{1}{2}$

Hence, $\vec{a}=-\frac{1}{2}(-10\hat{i}-7\hat{k}+8\hat{k})=5\hat{i}+\frac{7}{2}\hat{j}-4\hat{k}$

19 **(c)**

The projection of

$\vec{\mathbf{a}}$ on $\vec{\mathbf{b}}=\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}$

$=\frac{(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+5\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}})}{\sqrt{2^2+3^2+1^2}}=\frac{8}{\sqrt{14}}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	D	B	D	A	B	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	C	A	A	B	A	A	C	B