DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-VECTOR ALGEBRA

1 **(b)**

Given vectors $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $a\hat{i} + b\hat{j} + c\hat{k}$ will be perpendicular, if

$(2\hat{i} + 3\hat{j} - 4\hat{k}).(a\hat{i} + b\hat{j} + c\hat{k}) = 0 \Rightarrow 2a + 3b - c = 0$

Clearly, $a = 4, b = 4, c = 5$ satisfy the above equation

2 **(a)**

We have ,$\vec{\alpha} = x(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + y(\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + z(\vec{\mathbf{c}} \times \vec{\mathbf{a}})$

Taking dot product with $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ respectively, we get

$\vec{\alpha} \cdot \vec{\mathbf{a}} = y[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] \Rightarrow y = 8(\vec{\alpha} \cdot \vec{\mathbf{a}})$

$\vec{\alpha} \cdot \vec{\mathbf{b}} = z((\vec{\mathbf{c}} \times \vec{\mathbf{a}}) \cdot \vec{\mathbf{b}})$

$\Rightarrow \vec{\alpha} \cdot \vec{\mathbf{b}} = z[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] \Rightarrow z = 8(\vec{\alpha} \cdot \vec{\mathbf{b}})$

and $\vec{\alpha} \cdot \vec{\mathbf{c}} = x(\vec{\mathbf{a}} \times \vec{\mathbf{b}}. \vec{\mathbf{c}})$

$\vec{\alpha} \cdot \vec{\mathbf{c}} = x[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] \Rightarrow x = 8(\vec{\alpha} \cdot \vec{\mathbf{c}})$

$\therefore x + y + z = 8\vec{\alpha} \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$

3 **(d)**

Let $\vec{\mathbf{c}} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $\vec{\mathbf{d}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} - 15\hat{\mathbf{k}}$

For collinears, $\vec{\mathbf{c}} \times \vec{\mathbf{d}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & -5 \\ a & b & -15 \end{vmatrix} = \vec{\mathbf{0}}$

$\Rightarrow \hat{\mathbf{i}}(-15 + 5b) - \hat{\mathbf{j}}(-45 + 5a) + \hat{\mathbf{k}}(3b - a) = \vec{\mathbf{0}}$

$\Rightarrow -15 + 5b = 0, \quad -45 + 5a = 0, \quad 3b - a = 0$

$\Rightarrow b = 3, a = 9$

4 **(d)**

$|\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta$

$= 1^2 + 1^2 2 \cdot 1 \cdot 1 \cdot \cos 60° \quad [\because |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1]$

$= 2 - 2 \cdot \frac{1}{2} = 1$

5 **(c)**

Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}, \quad \vec{\mathbf{b}} = 2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$

Now take option (c).

Let $\vec{\mathbf{c}} = 0\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

Now, $\vec{a}\cdot(\vec{b}\times\vec{c})=\begin{vmatrix}1 & -2 & -3\\ 2 & 0 & 0\\ 0 & -4 & -6\end{vmatrix}$

$=1(0)+2(-12)-3(-8)=0$

6 **(a)**

$(\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})$

$=\vec{a}\times(\vec{a}\times\vec{b})+\vec{b}\times(\vec{a}\times\vec{b})$

$=(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}+(\vec{b}\cdot\vec{b})\vec{a}-(\vec{b}\cdot\vec{a})\vec{b}$

$=(\vec{a}\cdot\vec{b})\vec{a}-\vec{b}+\vec{a}-(\vec{b}\cdot\vec{a})\vec{b}$

$=(\vec{a}-\vec{b})(\vec{a}\cdot\vec{b}-1)$

$\therefore$ Given vector is parallel to $(\vec{a}-\vec{b})$.

7 **(a)**

$\overrightarrow{AB}=(2-1)\hat{i}+(0-2)\hat{j}+(3+1)\hat{k}$

$=\hat{i}-2\hat{j}+4\hat{k}$

and

$\overrightarrow{AC}=(3-1)\hat{i}+(-1-2)\hat{j}+(2+1)\hat{k}$

$=2\hat{i}-3\hat{j}+3\hat{k}$

$\cos\theta=\dfrac{(\hat{i}-2\hat{j}+4\hat{k})\cdot(2\hat{i}-3\hat{j}+3\hat{k})}{\sqrt{1+4+16}\sqrt{4+9+9}}$

$=\dfrac{2+6+12}{\sqrt{21}\sqrt{22}}=\dfrac{20}{\sqrt{462}}$

$\Rightarrow\sqrt{462}\cos\theta=20$

8 **(c)**

$[\vec{u}\,\vec{v}\,\vec{w}]=|\vec{u}\cdot(\vec{v}\times\vec{w})|$

$=|\vec{u}\cdot(3\hat{i}-7\hat{j}-\hat{k})|$

$=|\vec{u}|\sqrt{59}\cos\theta$

$\therefore$ Maximum value of $[\vec{u}\,\vec{v}\,\vec{w}]=\sqrt{59}$ $(\because|\vec{u}|=1,\cos\theta\le 1)$

10 **(b)**

Given, force $=5\left(\frac{2\hat{i}-2\hat{j}+\hat{k}}{|2\hat{i}-2\hat{j}+\hat{k}|}\right)=\frac{5}{3}(2\hat{i}-2\hat{j}+\hat{k})$

Displacement $=(5\hat{i}+3\hat{j}+7\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$

$=(4\hat{i}+\hat{j}+4\hat{k})$

$\therefore$ Required work done $=$ Force $\cdot$ Displacement

$=\frac{5}{3}[(2\hat{i}-2\hat{j}+\hat{k})\cdot(4\hat{i}+\hat{j}+4\hat{k})]$

$=\frac{5}{3}[8-2+4]=\frac{50}{3}$ unit

11 **(b)**

We know that the equation of the plane passing through three non-collinear points $\vec{a},\vec{b},\vec{c}$ is

$\vec{r}\cdot(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})=[\vec{a}\,\vec{b}\,\vec{c}]$

12 **(a)**

We have,

Required vector $\vec{r} = \lambda(\hat{a} + \hat{b})$, λ is a scalar

$\Rightarrow \vec{r} = \lambda\left\{\frac{1}{9}(7\hat{i} - 4\hat{j} - 4\hat{k}) + \frac{1}{3}(-2\hat{i} - \hat{j} + 2\hat{k})\right\} = \frac{\lambda}{9}(\hat{i} - 7\hat{j} + 2\hat{k})$

Now,

$|\vec{r}| = 3\sqrt{6} \Rightarrow |\vec{r}|^2 = 54 \Rightarrow \frac{\lambda^2}{81}(1 + 49 + 4) = 54 \Rightarrow \lambda = \pm 9$

Hence, required vector $\vec{r} = \pm(\hat{i} - 7\hat{j} + 2\hat{k})$

Clearly, option (a) is true for $\lambda = 1$

13 **(b)**

Given vectors are collinear, if $\begin{vmatrix} 2 & 1 & 1 \\ 6 & -1 & 2 \\ 14 & -5 & p \end{vmatrix} = 0$

$\Longrightarrow 2[-p + 10] - 1[6p - 28] + 1[-30 + 14] = 0$

$\Longrightarrow -8p + 32 = 0$

$\Longrightarrow p = 4$

14 **(d)**

Given,

$\frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}} = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$

$\therefore \frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}} = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})\vec{\mathbf{a}}$

On comparing the coefficient of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, we get

$\frac{1}{2}|\vec{\mathbf{b}}||\vec{\mathbf{c}}| = -\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 0$

$\Longrightarrow \frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}| = -|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\theta \Longrightarrow \cos\theta = -\frac{1}{3}$

$\Longrightarrow 1 - \sin^2\theta = \frac{1}{9} \Longrightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

15 **(c)**

Let $\vec{\mathbf{A}} = 7\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$, $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $\vec{\mathbf{C}} = -4\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

Now, $\overrightarrow{\mathbf{AB}} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, $\overrightarrow{\mathbf{BC}} = -3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

and $\overrightarrow{\mathbf{CA}} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

Here, $|\overrightarrow{\mathbf{AB}}| = |\overrightarrow{\mathbf{BC}}| = 3\sqrt{2}$ and $|\overrightarrow{\mathbf{CA}}| = 6$

Now, $|\overrightarrow{\mathbf{AB}}|^2 + |\overrightarrow{\mathbf{BC}}|^2 = |\overrightarrow{\mathbf{AC}}|^2$

Hence, the triangle is right angled isosceles triangle.

16 **(b)**

We know that if A and B are two points and P is any point on AB. Then,

$m\,P\vec{A} + n\,P\vec{B} = (m + n)P\vec{C}$, where C divides AB in the ratio $n{:}m$

Here, $m = n = 1$

$\therefore P\vec{A} + P\vec{B} = 2P\vec{C}$

17 **(a)**

$(2\vec{\mathbf{a}} + 3\vec{\mathbf{b}}) \times (5\vec{\mathbf{a}} + 7\vec{\mathbf{b}}) + \vec{\mathbf{a}} \times \vec{\mathbf{b}}$

$= \vec{\mathbf{0}} + 14(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) - 15(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + \vec{\mathbf{0}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}$

$= \vec{\mathbf{0}}$

19 **(c)**

Let $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\overrightarrow{\mathbf{OB}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{OC}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\therefore a = |\overrightarrow{\mathbf{OA}}| = \sqrt{6}, b = |\overrightarrow{\mathbf{OB}}| = \sqrt{35}$

and $\vec{\mathbf{c}}|\overrightarrow{\mathbf{OC}}| = \sqrt{41}$

$$\therefore \cos A = \frac{b^2 + c^2 + a^2}{2bc}$$

$$= \frac{(\sqrt{35})^2 + (\sqrt{41})^2 - (\sqrt{6})^2}{2\sqrt{35}\sqrt{41}}$$

$$\Rightarrow \cos A = \sqrt{\frac{35}{41}}$$

$$\Rightarrow \sin^2 A = \frac{35}{41}$$

20 **(d)**

Let $\vec{p} \neq \vec{0}$. Then,

$\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar, which is a contradiction

Hence, $\vec{r} = \vec{0}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	D	D	C	A	A	C	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	D	C	B	A	A	C	D