DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :4

Topic :-VECTOR ALGEBRA

1 **(c)**

We have,

$$\left.\begin{array}{l}\vec{r}\cdot\vec{a}=0\Rightarrow\vec{r}\perp\vec{a}\\ \vec{r}\cdot\vec{b}=0\Rightarrow\vec{r}\perp\vec{b}\\ \vec{r}\cdot\vec{c}=0\Rightarrow\vec{r}\perp\vec{c}\end{array}\right\}\Rightarrow\vec{a},\vec{b},\vec{c}\text{ are coplanar}$$

Hence, $[\vec{a}\vec{b}\vec{c}]=0$

2 **(b)**

$$\cos\frac{\pi}{3}=\frac{(\hat{\mathbf{i}}+\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+a\hat{\mathbf{k}})}{\sqrt{2}\sqrt{1+1+a^2}}$$

$$\Rightarrow\frac{1}{2}=\frac{1+a}{\sqrt{2}\sqrt{2+a^2}}$$

$$\Rightarrow\frac{1}{4}=\frac{(1+a)^2}{2(2+a^2)}$$

$\Rightarrow 2+a^2=2(1+a^2+2a)$

$\Rightarrow a^2+4a=0$

$\Rightarrow a=0,\ -4$

3 **(b)**

Let the required vector be $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$

It makes equal angles with the unit vectors

$\vec{b}=\frac{1}{3}(\hat{i}-2\hat{j}+2\hat{k}),\vec{c}=\frac{1}{5}(-4\hat{i}-3\hat{k})$ and $\vec{d}=\hat{j}$

$\therefore\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=\vec{a}\cdot\vec{d}\ [\because\vec{b},\vec{c},\vec{d}$ are unit vectors$]$

$$\Rightarrow\frac{1}{3}(x-2y+2z)=\frac{1}{5}(-4x-3z)=y$$

$\Rightarrow x-2y+2z=3y$ and $-4x-5y-3z=0$

$\Rightarrow x-5y+2z=0$ and $4x+5y+3z=0$

$$\Rightarrow\frac{x}{-5}=\frac{y}{1}=\frac{z}{5}=\lambda\text{ (say)}$$

$\Rightarrow x=-5\lambda, y=\lambda, z=5\lambda$ for some scalar λ

$\Rightarrow\vec{a}=\lambda(-5\hat{i}+\hat{j}+5\hat{k})$

Clearly, option (b) is true for $\lambda=1$

4 **(d)**

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(4+2) - \hat{\mathbf{j}}(4-1) + \hat{\mathbf{k}}(-4-2)$$

$$= 6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$$

$$\Rightarrow |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{36+9+36} = \sqrt{81} = 9$$

∴ Required vectors are

$$\pm 6\left|\frac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|}\right|$$

$$= \pm \frac{6}{9}\left(6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}\right)$$

$$= \pm 2(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

6 **(d)**

(a) Let $\vec{\mathbf{p}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ where at least one of x, y, z is non-zero. Let

$\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{l}} + a_3\hat{\mathbf{k}}$

$\vec{\mathbf{b}} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{l}} + b_3\hat{\mathbf{k}}$

$\vec{\mathbf{c}} = c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{l}} + c_3\hat{\mathbf{k}}$

∴ By given conditions

$a_1x + a_2y + a_3z = 0$

$b_1x + b_2y + b_3z = 0$

$c_1x + c_2y + c_3z = 0$

$$\Rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_3 & c_3 \end{vmatrix} = 0$$

$\Rightarrow [\,\vec{\mathbf{a}}\;\vec{\mathbf{b}}\;\vec{\mathbf{c}}\,] = 0$

$\Rightarrow \vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar.

(b) Vectors are coplanar, if

$$\begin{vmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

ie, $-7 = 0$

Which is not possible.

(c) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}}$

$\Rightarrow \vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ is coplanar with $\vec{b}$ and $\vec{c}$.

(d) $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1$

$\therefore |\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = (\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}})$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$

$= 1 + 1 = 2 \cdot 1 \cdot 1 \cos\frac{\pi}{3} = 3$

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3} > 1$

7 **(d)**

Here, $\vec{\mathbf{a}}_1 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}}, \quad \vec{\mathbf{a}}_2 = -2\hat{\mathbf{i}} + 7\hat{\mathbf{k}}$

$\vec{\mathbf{b}}_1 = -4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}_2 = -4\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now, $\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1 = \hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

and

$$\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix} = \hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$\Rightarrow \left|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2\right| = \sqrt{1 + 16 + 64} = 9$$

Now,

$$(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot (\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2) = (\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 8\hat{\mathbf{k}})$$

$$= 1 + 24 + 56 = 81$$

$\therefore$ Shortest distance,

$$d = \left|\frac{(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot (\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2)}{\left|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2\right|}\right|$$

$$= \frac{81}{9} = 9 \text{ unit}$$

8 **(b)**

We know that a vector perpendicular to the plane containing the points $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ is given by $\vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{B}} \times \vec{\mathbf{C}} + \vec{\mathbf{C}} \times \vec{\mathbf{A}}$.

Given, $\vec{\mathbf{A}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\vec{\mathbf{C}} = 0\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now,

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} \times \vec{\mathbf{C}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

$$\vec{\mathbf{C}} \times \vec{\mathbf{A}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

Thus,

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{B}} \times \vec{\mathbf{C}} + \vec{\mathbf{C}} \times \vec{\mathbf{A}}$$

$$= (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

$$= 8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

9 **(c)**

Given,

$$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = \frac{1}{4}$$

$\Rightarrow(|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta)^2 = \frac{1}{4}$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \theta = \frac{\pi}{6}$

10 **(b)**

Given that, $|\vec{\mathbf{a}}| = 3$, $|\vec{\mathbf{b}}| = 4$ and $\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}$ is perpendicular to $\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}$.

$\therefore (\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}) = 0$

$\Rightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{a}} - \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\lambda + \lambda\vec{\mathbf{b}} \cdot \vec{\mathbf{a}} - \lambda^2\vec{\mathbf{b}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow |\vec{\mathbf{a}}|^2 - \lambda^2|\vec{\mathbf{b}}|^2 = 0$

$\Rightarrow \lambda^2 = \frac{|\vec{\mathbf{a}}|^2}{|\vec{\mathbf{b}}|^2} \Rightarrow \lambda = \frac{|\vec{\mathbf{a}}|}{|\vec{\mathbf{b}}|} = \frac{3}{4}$

11 **(a)**

$(\vec{\mathbf{x}} - \vec{\mathbf{y}}) \times (\vec{\mathbf{x}} + \vec{\mathbf{y}})$

$= \vec{\mathbf{x}} \times \vec{\mathbf{x}} + \vec{\mathbf{x}} \times \vec{\mathbf{y}} - \vec{\mathbf{y}} \times \vec{\mathbf{x}} - \vec{\mathbf{y}} \times \vec{\mathbf{y}}$

$= \vec{\mathbf{0}} + \vec{\mathbf{x}} \times \vec{\mathbf{y}} + \vec{\mathbf{x}} \times \vec{\mathbf{y}} - \vec{\mathbf{0}}$

$= 2(\vec{\mathbf{x}} \times \vec{\mathbf{y}})$

12 **(a)**

$\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 0$

$\Rightarrow \lambda - 1 + 2\mu = 0$

$\Rightarrow \lambda + 2\mu = 1$ ...(i)

$\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 0$

$\Rightarrow 2\lambda + 4 + \mu = 0$

$\Rightarrow 2\lambda + \mu = -4$....(ii)

On solving Eqs. (i) and (ii), we get

$\lambda - 3, \mu = 2$

15 **(b)**

The projection $\vec{x} \times \vec{y}$ on $\vec{z}$ is given by

$$\frac{(\vec{x} \times \vec{y}) \cdot \vec{z}}{|\vec{z}|} = \frac{1}{|\vec{z}|}[\vec{x}\ \vec{y}\ \vec{z}] = \frac{1}{13}\begin{vmatrix} 3 & -6 & -1 \\ 1 & 4 & -3 \\ 3 & -4 & -12 \end{vmatrix} = -14$$

16 **(c)**

We have,

$\vec{a} \times \{\vec{a} \times \{\vec{a} \times (\vec{a} \times \vec{b})\}\}$

$= \vec{a} \times \{\vec{a} \times \{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\}\} = \vec{a} \times \{\vec{0} - |\vec{a}|^2(\vec{a} \times \vec{b})\}$

$= -|\vec{a}|^2\{\vec{a} \times (\vec{a} \times \vec{b})\} = -|\vec{a}|^2\{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\}$

$= -|\vec{a}|^2\{0 - |\vec{a}|^2\vec{b}\} = |\vec{a}|^4\vec{b}$

19 **(c)**

For an abtuse angle

$(cx\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (x\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2cx\hat{\mathbf{k}}) < 0$

$\Rightarrow cx^2 - 12 + 6cx < 0$

$\Rightarrow cx^2 + 6cx - 12 < 0$

$\therefore (6c)^2 - 4c(-12) < 0 \quad [\because f(x) < 0 \Rightarrow D < 0]$

$\Rightarrow 36c\left(c + \frac{4}{3}\right) < 0$

$\Rightarrow -\frac{4}{3} < c < 0$

20 **(a)**

$\cos\theta = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$

$= \frac{\left(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) \cdot (6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{\sqrt{2^2 + 2^2 + (-1)^2}\sqrt{6^2 + (-3)^2 + 2^2}}$

$= \frac{12 - 6 - 2}{\sqrt{4 + 4 + 1}\sqrt{36 + 9 + 4}} = \frac{4}{21}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	D	C	D	D	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	C	C	B	C	A	B	C	A