DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-VECTOR ALGEBRA

1 **(a)**

Taking A as the origin, let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively

$\therefore \vec{B}E + \vec{A}F = \left(\frac{\vec{c}}{2} - \vec{b}\right) + \left(\frac{\vec{b} + \vec{c}}{2} - \vec{0}\right) = \vec{c} - \frac{\vec{b}}{2} = \vec{D}C$

2 **(a)**

Since, $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors.

$\Rightarrow |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1$

and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$ .....(i)

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2 = (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}})$

$= 1 + 1 + 1 + 0 = 3$ [from Eq. (i)]

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}| = \sqrt{3}$

3 **(c)**

Any vector lying in the plane of $\vec{a}$ and $\vec{b}$ is of the from $x\vec{a} + y\vec{b}$

It is given that $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$

$\therefore \vec{c} = \lambda(x\vec{a} + y\vec{b})$ for some scalar λ

$\Rightarrow d\hat{i} + \hat{j} + (2d-1)\hat{k} = \lambda\{x(\hat{i} - 2\hat{j} + 3\hat{k}) + y(3\hat{i} + 3\hat{j} - \hat{k})\}$

$\Rightarrow d\hat{i} + \hat{j} + (2d-1)\hat{k} = \lambda\{(x+3y)\hat{i} + (-2x+3y)\hat{j} + (3x-y)\hat{k}\}$

$\Rightarrow \lambda(x+3y) = d, \lambda(-2x+3y) = 1$ and $\lambda(3x-y) = (2d-1)$

$[\because \hat{i}, \hat{j}, \hat{k}$ are non $-$ coplanar$]$

Solving $\lambda(x+3y) = d$ and $3x - y = 2d - 1$, we get

$x = \frac{7d-3}{10\lambda}$ and $y = \frac{d+1}{10\lambda}$

Substituting these values in $\lambda(x+3y) = d$, we get $11d = -1$

ALTER *clearly, $\vec{c}$ is perpendicular to $\vec{a} \times \vec{b}$*

$\therefore \vec{c} \cdot (\vec{a} \times \vec{b}) = 0$

$$\Rightarrow [\vec{c}\vec{a}\vec{b}] = 0 \Rightarrow \begin{vmatrix} d & 1 & 2d-1 \\ 1 & -2 & 3 \\ 3 & 3 & -1 \end{vmatrix} = 0 \Rightarrow 11d = -1$$

4 **(c)**

$\because$ $\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{r}}$ are reciprocal vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ respectively.

$\therefore \vec{\mathbf{p}} \cdot \vec{\mathbf{a}} = 1, \vec{\mathbf{p}} \cdot \vec{\mathbf{b}} = 0, \vec{\mathbf{p}} \cdot \vec{\mathbf{c}}$ etc.

$\therefore (l\vec{\mathbf{a}} + m\vec{\mathbf{b}} + n\vec{\mathbf{c}}) \cdot (l\vec{\mathbf{p}} + m\vec{\mathbf{q}} + n\vec{\mathbf{r}}) = l^2 + m^2 + n^2$

5 **(b)**

Given expression $= 2(1 + 1 + 1) - 2\sum(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$

$= 6 - 2\sum(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$ ...(i)

But $(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})^2 \geq 0$

$\therefore (1 + 1 + 1) + 2\sum \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$

$\therefore 3 \geq -2\sum \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$ ...(ii)

From relations (i) and (ii), we get

Given expression $\leq 6 + 3 = 9$

6 **(a)**

Let $\overrightarrow{\mathbf{OA}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$\therefore \overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ work don, $W = \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{AB}}$

$= (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= 4 - 6 + 4 = 2$

7 **(d)**

$\overrightarrow{\mathbf{AC}} = (a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = (a - 2)\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

and $\overrightarrow{\mathbf{BC}} = (a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) = (a - 1)\hat{\mathbf{i}} + 6\hat{\mathbf{k}}$

Since, the ΔABC is right angled at C, then

$\overrightarrow{\mathbf{AC}} \cdot \overrightarrow{\mathbf{BC}} = 0$

$\Rightarrow \{(a - 2)\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\} \cdot \{(a - 1)\hat{\mathbf{i}} + 6\hat{\mathbf{k}}\} = 0$

$\Rightarrow (a - 2)(a - 1) = 0 \Rightarrow a = 1$ and 2

8 **(a)**

We have,

$(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$

$\Leftrightarrow -\vec{c} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\vec{b} \times \vec{c})$

$\Leftrightarrow -\{(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}\} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$\Leftrightarrow (\vec{a} \cdot \vec{b})\vec{c} - (\vec{c} \cdot \vec{b})\vec{a} = 0$

$\Leftrightarrow (\vec{b} \cdot \vec{a})\vec{c} - (\vec{b} \cdot \vec{c})\vec{a} = 0$

$\Leftrightarrow \vec{b} \times (\vec{c} \times \vec{a}) = 0$

9 **(b)**

Clearly,

$(\vec{a} + \vec{b}) \times \{\vec{c} - (\vec{a} + \vec{b})\}$

$= (\vec{a} + \vec{b}) \times \vec{c} - (\vec{a} + \vec{b}) \times (\vec{a} + \vec{b}) = (\vec{a} + \vec{b}) \times \vec{c}$

10 **(a)**

$\overrightarrow{\mathbf{PQ}} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$

and $\vec{\mathbf{F}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\therefore$ Moment $= |\overrightarrow{\mathbf{PQ}} \times \vec{\mathbf{F}}|$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{vmatrix}$$

$= -2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$Magnitude of moment $= \sqrt{4 + 49 + 4} = \sqrt{57}$

11 **(b)**

Since, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3}$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 3$

$\Longrightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \frac{1}{2}$ .....(i)

$\because [|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1$, given]

$\therefore (3\vec{\mathbf{a}} - 4\vec{\mathbf{b}}) \cdot (2\vec{\mathbf{a}} + 5\vec{\mathbf{b}}) = 6 + 7\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - 20$

$= 6 + \frac{7}{2} - 20$

$= -\frac{21}{2}$ [from Eq.(i)]

12 **(c)**

We have,

$\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$

$\Rightarrow (\hat{a} \cdot \hat{c})\hat{b} - (\hat{a} \cdot \hat{b})\hat{c} = \frac{1}{2}\hat{b}$

$\Rightarrow \left\{(\hat{a} \cdot \hat{c}) - \frac{1}{2}\right\}\hat{b} - (\hat{a} \cdot \hat{b})\hat{c} = 0$

$\Rightarrow \hat{a} \cdot \hat{c} - \frac{1}{2} = 0$ and $\hat{a} \cdot \hat{b} = 0$ $\left[\begin{matrix} \because \hat{b}, \hat{c} \\ \text{are non} - \text{collinear vectors} \end{matrix}\right]$

$\Rightarrow \cos\theta = \frac{1}{2}$, where θ is the angle between $\hat{a}$ and $\hat{c}$

$\Rightarrow \theta = \pi/3$

14 **(b)**

The given line is parallel to the vector $\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$.The required plane passing through the point (2, 3, 1)*ie*, $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and is perpendicular to the vector

$\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Its equation is

$[\left(\vec{\mathbf{r}} - \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\right] \cdot \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) = 0$

$\Longrightarrow \vec{\mathbf{r}} \cdot \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) = 1$

15 **(c)**

$(\vec{\mathbf{a}} - \vec{\mathbf{b}}) \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}} - \vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) - \vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] - [\vec{\mathbf{b}}\ \vec{\mathbf{c}}\ \vec{\mathbf{a}}] = 0$

16 **(a)**

We have,

$|\hat{n}_1+\hat{n}_2|^2=|\hat{n}_1|+|\hat{n}_2|+2\hat{n}_1\cdot\hat{n}_2$

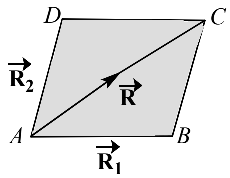
$\Rightarrow|\hat{n}_1+\hat{n}_2|^2=|\hat{n}_1|^2+|\hat{n}_2|^2+2|\hat{n}_1|+|\hat{n}_2|\cos\theta$

$\Rightarrow|\hat{n}_1+\hat{n}_2|^2=1+1+2\cos\theta\ =4\cos^2\frac{\theta}{2}$

$\therefore\cos\frac{\theta}{2}=\frac{1}{2}|\hat{n}_1+\hat{n}_2|$

17 **(d)**

Let $\vec{\mathbf{R}}_1=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}$



and $\vec{\mathbf{R}}_2=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{R}}$ (along $\overrightarrow{\mathbf{AC}}$) $=\vec{\mathbf{R}}_1+\vec{\mathbf{R}}_2$

$=3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{a}}$ (unit vector along AC) $=\frac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|}$

$=\frac{3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{\sqrt{9+36+4}}$

$=\frac{1}{7}(3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

18 **(a)**

Let $P(60\hat{\mathbf{i}}+3\hat{\mathbf{j}})$, $Q(40\hat{\mathbf{i}}-8\hat{\mathbf{j}})$and $R(a\hat{\mathbf{i}}-52\hat{\mathbf{j}})$ be the collinear points. Then $\overrightarrow{\mathbf{PQ}}=\lambda\overrightarrow{\mathbf{QR}}$ for some scalar λ

$\Rightarrow(-20\hat{\mathbf{i}}-11\hat{\mathbf{j}})=\lambda[(a-40)\hat{\mathbf{i}}-44\hat{\mathbf{j}}]$

$\Longrightarrow\lambda(a-40)=-20,-44\lambda=-11$

$\Longrightarrow\lambda(a-40)=-20,\lambda=\frac{1}{4}$

$\therefore a-40=-20\times4\Longrightarrow a=-40$

19 **(a)**

We have,

$\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}$ and $\vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$

$\Rightarrow\vec{a}+\vec{b}+\vec{c}+\vec{d}=(\alpha+1)\vec{d}$ and $\vec{a}+\vec{b}+\vec{c}+\vec{d}=(\beta+1)\vec{a}$

$\Rightarrow(\alpha+1)\vec{d}=(\beta+1)\vec{a}$

If $\alpha\neq-1$, then

$(\alpha+1)\vec{d}=(\beta+1)\vec{a}\Rightarrow\vec{d}=\frac{\beta+1}{\alpha+1}\vec{a}$

$\therefore\vec{a}+\ \vec{b}+\ \vec{c}=\alpha\vec{d}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = \alpha\left(\frac{\beta+1}{\alpha+1}\right)\vec{a}$

$\Rightarrow \left\{1 - \frac{\alpha(\beta+1)}{\alpha+1}\right\}\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar

It is a contradiction to the given condition

$\therefore \alpha = -1 \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$

20 **(c)**

Let the unit vector $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ is perpendicular to $\hat{\mathbf{i}} - \hat{\mathbf{j}}$, then we get

$$\frac{(\hat{\mathbf{i}}+\hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}}-\hat{\mathbf{j}})}{\sqrt{2}} = \frac{1-1}{\sqrt{2}} = 0$$

$\therefore \frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ is the unit vector

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	C	B	A	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	C	B	C	A	D	A	A	C