DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-VECTOR ALGEBRA

1 **(d)**

$(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot(\vec{\mathbf{b}}+\vec{\mathbf{c}})\times(\vec{\mathbf{c}}+\vec{\mathbf{a}})$

$=(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot[\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{b}}\times\vec{\mathbf{a}}+\vec{\mathbf{c}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}]$

$=\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})$

$=\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$

$=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]-[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=0$

2 **(b)**

$\because$ $\vec{\mathbf{a}}$,$\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar vectors, so $2\vec{\mathbf{a}}-\vec{\mathbf{b}}$ $2\vec{\mathbf{b}}-\vec{\mathbf{c}}$ and $2\vec{\mathbf{c}}-\vec{\mathbf{a}}$ are also coplanar. Thus

$[2\vec{\mathbf{a}}-\vec{\mathbf{b}}\;\;2\vec{\mathbf{b}}-\vec{\mathbf{c}}\;\;2\vec{\mathbf{c}}-\vec{\mathbf{a}}]=0$

3 **(b)**

Clearly, angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ $=\frac{\pi}{2}$

$\Rightarrow\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0$

$\therefore|\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2=a^2+b^2+2\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$

$=1+1+0=2$

$\Rightarrow|\vec{\mathbf{a}}+\vec{\mathbf{b}}|=\sqrt{2}$

5 **(d)**

Given, $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}=-\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$

$\Longrightarrow(\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}-(\vec{\mathbf{c}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}=-\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$

On comparing both sides, we get

$(\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}=0$

$|\vec{\mathbf{c}}|\vec{\mathbf{a}}|\cos\theta=0$

$\Longrightarrow\theta=\frac{\pi}{2}$

6 **(c)**

Now, $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 1 & 1\\ 1 & 1 & 0\end{vmatrix}$

$=\hat{\mathbf{i}}(-1)+\hat{\mathbf{j}}(1)+\hat{\mathbf{k}}(0)=-\hat{\mathbf{i}}+\hat{\mathbf{j}}$

and $|(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})|=\sqrt{1^2+1^2}=\sqrt{2}$

Vector perpendicular to both of the vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}$

$$=\frac{(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})}{|(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})|}$$

$$=\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}=\frac{-1}{\sqrt{2}}(\hat{\mathbf{i}}-\hat{\mathbf{j}})$$

$=c(\hat{\mathbf{i}}-\hat{\mathbf{j}})$, c is a scalar.

7 **(b)**

It is given that $(\vec{a}+\vec{b})||\vec{c}$ and $(\vec{c}+\vec{a})||\vec{b}$

$\therefore (\vec{a}+\vec{b})\times\vec{c}=0$ and $(\vec{c}+\vec{a})\times\vec{b}=0$

$\Rightarrow \vec{a}\times\vec{c}+\vec{b}\times\vec{c}=0$ and $\vec{c}\times\vec{b}+\vec{a}\times\vec{b}=0$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Hence, $\vec{a},\vec{b},\vec{c}$ form the sides of a triangle

8 **(a)**

$\because$ Displacement, $\overrightarrow{\mathbf{AB}}=(3-2)\hat{\mathbf{i}}+(1+1)\hat{\mathbf{j}}+(2-1)\hat{\mathbf{k}}$

$=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

and force, $\vec{\mathbf{F}}=\frac{\sqrt{6}(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})}{\sqrt{6}}$

$=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$

$\therefore$ Work done $=\vec{\mathbf{F}}\cdot\overrightarrow{\mathbf{AB}}=(1+2\hat{\mathbf{j}}+\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})=6$

9 **(c)**

let $\vec{\mathbf{a}}=l\hat{\mathbf{i}}+m\hat{\mathbf{j}}+n\hat{\mathbf{k}}$ makes an angle $\frac{\pi}{4}$ with z-axis

Also, $l^2+m^2+n^2=1$

Here, $n=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}},\ l^2+m^2=\frac{1}{2}$ ......(i)

$\therefore\ \vec{\mathbf{a}}=l\hat{\mathbf{i}}+m\hat{\mathbf{j}}+\frac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Rightarrow \vec{\mathbf{a}}+\hat{\mathbf{i}}+\hat{\mathbf{j}}=(l+1)\hat{\mathbf{i}}(m+1)\hat{\mathbf{j}}+\frac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Rightarrow |\vec{\mathbf{a}}+\hat{\mathbf{i}}+\hat{\mathbf{j}}|=\sqrt{(l+1)^2+(m+1)^2+\left(\frac{1}{\sqrt{2}}\right)^2}$

$\Rightarrow 1=l^2+m^2+2+2l+2m+\frac{1}{2}$

$\Rightarrow l+m=-1$ (From Eq. (i)

$\Rightarrow l^2+m^2+2lm=1$

$\Rightarrow 2lm=\frac{1}{2}$

$\Rightarrow l=m=-\frac{1}{2}$

$\left(\because l=m=\frac{1}{2}\right.$, is not satisfied the given equition$\left.\right)$

$\therefore\ \vec{\mathbf{a}} = -\frac{\hat{\mathbf{i}}}{2} - \frac{\hat{\mathbf{j}}}{2} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

10 **(b)**

Given, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 + |\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2 = 144$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2(\sin^2\theta + \cos^2\theta) = 144$

$\Longrightarrow 16|\vec{\mathbf{b}}|^2 = 144$

$\Longrightarrow |\vec{\mathbf{b}}| = 3$

11 **(c)**

Since, $m\,\vec{\mathbf{a}}$ is a unit vector, if and only, if

$|m\,\vec{\mathbf{a}}| = 1 \Rightarrow |m\,||\vec{\mathbf{a}}| = 1 \Rightarrow m|\vec{\mathbf{a}}| = 1$

$\Rightarrow m = \frac{1}{|\vec{\mathbf{a}}|}$

12 **(b)**

Resultant force $\vec{F}$ is given by

$\vec{F} = (2\hat{i} - 5\hat{j} + 6\hat{k}) - (-\hat{i} + 2\hat{j} - \hat{k}) = \hat{i} - 3\hat{j} + 5\hat{k}$

Let $\vec{d}$ be the displacement vector. Then,

$\vec{d} = \vec{AB}$

$\Rightarrow \vec{d} = (6\hat{i} + \hat{j} - 3\hat{k}) - (4\hat{i} - 3\hat{j} - 2\hat{k}) = 2\hat{i} + 4\hat{j} - \hat{k}$

$\therefore W =$ Work done

$\Rightarrow W = \vec{F} \cdot \vec{d}$

$\Rightarrow W = (\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (2\hat{i} + 4\hat{j} - \hat{k})$

$\Rightarrow W = 2 - 12 - 5 = -15$ units

13 **(d)**

Since, P, Q, R are collinear. Therefore,

$\vec{PQ} = m\,\vec{QR}$ for same scalar m

$\Rightarrow -2\hat{j} = m\left[(a-1)\hat{i} + (\vec{b}+1)\hat{j} + c\,\hat{k}\right]$ for some non-zero scalar m

$\Rightarrow (a-1)m = 0,\ (b+1)m = -2,\ cm = 0$

$\Rightarrow a = 1,\ c = 0,\ b \in R$

14 **(b)**

The direction cosines of a vector making equal angles with the coordinate axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Therefore, the unit vector along the vector making equal angles with the coordinate axes is

$\vec{b} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \vec{a} \cdot \vec{b}$

$= (4\hat{i} - 3\hat{j} + 2\hat{k}) \cdot \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) = \frac{4-3+2}{\sqrt{3}} = \sqrt{3}$

15 **(a)**

$[2\,\hat{\mathbf{i}}\ 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}\,]$

$= -30\,[\,\hat{\mathbf{i}}\ \hat{\mathbf{j}}\ \hat{\mathbf{k}}\,]$

$= -30 \quad (\because [\,\hat{\mathbf{i}}\ \hat{\mathbf{j}}\ \hat{\mathbf{k}}\,] = 1\,)$

16 **(b)**

We have,
$(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})\cdot\vec{d}$
$=\{((\vec{a}\times\vec{b})\cdot\vec{c})\vec{a}-((\vec{a}\times\vec{b})\cdot\vec{a})\vec{c}\}\cdot\vec{d}$
$=\{[\vec{a}\,\vec{b}\,\vec{c}]\vec{a}-0\}\cdot\vec{d}=[\vec{a}\,\vec{b}\,\vec{c}](\vec{a}\cdot\vec{d})$

17 **(d)**

Resultant force $\vec{\mathbf{F}}=\left(2\hat{\mathbf{i}}-5\hat{\mathbf{j}}+6\hat{\mathbf{k}}\right)+\left(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)$
$=\hat{\mathbf{i}}-3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$
and displacement, $\vec{\mathbf{d}}=\left(6\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)-\left(4\hat{\mathbf{i}}-3\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)$
$=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$
$\therefore$ work done $W=\vec{\mathbf{F}}\cdot\vec{\mathbf{d}}$
$=\left(\hat{\mathbf{i}}-3\hat{\mathbf{j}}+5\hat{\mathbf{k}}\right)\cdot\left(2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)$
$=-15$
$=15$ units [neglecting – ve sign]

18 **(a)**

The resultant force is given by
$$\vec{F}=6\frac{(\hat{i}-2\hat{j}+2\hat{k})}{\sqrt{1+4+4}}+7\frac{(2\hat{i}-3\hat{j}-6\hat{k})}{\sqrt{4+9+36}}=4\hat{i}-7\hat{j}-2\hat{k}$$
$\vec{d}=$ Displacement $=\vec{P}Q$
$\vec{d}=(5\hat{i}-\hat{j}+\hat{k})-(2\hat{i}-\hat{j}-3\hat{k})=3\hat{i}+4\hat{k}$
$\therefore$ Work done $=\vec{F}\cdot\vec{d}=12+0-8=4$ units

19 **(c)**

We know, $[\vec{\mathbf{b}}\times\vec{\mathbf{c}}\;\;\vec{\mathbf{c}}\times\vec{\mathbf{a}}\;\;\vec{\mathbf{a}}\times\vec{\mathbf{b}}]$
$=(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\cdot[(\vec{\mathbf{c}}\times\vec{\mathbf{a}})\cdot(\vec{\mathbf{a}}\times\vec{\mathbf{b}})]$
$=(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\cdot[\,((\vec{\mathbf{c}}\times\vec{\mathbf{a}})\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}-((\vec{\mathbf{c}}\times\vec{\mathbf{a}})\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}\,]$
$=(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\cdot(\,[\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,]\vec{\mathbf{a}}-[\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}\,\vec{\mathbf{a}}\,]\,\vec{\mathbf{b}})$
$=(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\cdot\vec{\mathbf{a}}[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\vec{\mathbf{c}}\,]-0$
$=[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,][\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]$
$=[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]^2$

20 **(d)**

$\because$ $\overrightarrow{\mathbf{QP}}$ is parallel to $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{DC}}$.
$\therefore$ $\overrightarrow{\mathbf{AB}}+\overrightarrow{\mathbf{DC}}=\overrightarrow{\mathbf{QP}}+\overrightarrow{\mathbf{QP}}=2\overrightarrow{\mathbf{QP}}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	B	B	B	D	C	B	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	A	B	D	A	C	D