DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :10

Topic :-VECTOR ALGEBRA

1 **(c)**

By the properties of midpoint theorem,

$\overrightarrow{\mathbf{PA}}+\overrightarrow{\mathbf{PB}}=2\overrightarrow{\mathbf{PC}}$

2 **(a)**

The vector equation of line passing through points (3, 2, 1) and (−2, 1, 3)

$\vec{\mathbf{r}}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}+\lambda\left[(-2-3)\hat{\mathbf{i}}+(1-2)\hat{\mathbf{j}}+(3-1)\hat{\mathbf{k}}\right]$

$=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}+\lambda(-5\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

3 **(d)**

$\because\quad \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\frac{5\pi}{6}$

$=-\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2}$

Since, the projection of $\vec{\mathbf{a}}$ in the direction of

$\vec{\mathbf{b}}=-\frac{6}{\sqrt{3}}$

$\Rightarrow-\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2|\vec{\mathbf{b}}|}=-\frac{6}{\sqrt{3}}$

$\Rightarrow|\vec{\mathbf{a}}|=\frac{6\times2}{3}=4$

4 **(d)**

Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ in $OXYZ$ system

Also, let $\vec{r}=X\hat{i}+Y\hat{j}+Z\hat{k}$ in the new coordinate system

Since the right handed rectangular system $OXYZ$ is rotated about z-axis through $\frac{\pi}{4}$ in anticlockwise direction. Therefore,

$x=X\cos\theta-Y\sin\theta$ and $y=X\sin\theta+Y\cos\theta$

$\Rightarrow x=X\cos\frac{\pi}{4}-Y\sin\frac{\pi}{4}, y=X\sin\frac{\pi}{4}+Y\cos\frac{\pi}{4}$

and, $z=Z$

It is given that $X=2\sqrt{2}, Y=3\sqrt{2}$ and $Z=4$

$\therefore x=2-3=-1, y=5$ and $z=4$

Hence, $\vec{r}=-\hat{i}+5\hat{j}+4\hat{k}$

ALITER Let $l_1, m_1, n_1; l_2, m_2, n_2$ and l_3, m_3, n_3 be the direction cosines of the new axes with respect to the old axes. Then,

$$l_1 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, m_1 = \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, n_1 = \cos\frac{\pi}{2} = 0$$

$$l_2 = \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}, m_2 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, n_2 = \cos\frac{\pi}{2} = 0$$

$$l_3 = \cos\frac{\pi}{2} = 0, m_3 = \cos\frac{\pi}{2} = 0, n_3 = \cos 0 = 1$$

Let x', y', z' and x, y, z be the components of the given vector with respect to new and old axes. Then,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} \\ 3\sqrt{2} \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -3 & +0 \\ 2 & +3 & +0 \\ 0 & 0 & +4 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}$$

Hence, the components of $\vec{a}$ in the $Oxyz$ coordinate system are $-1, 5, 4$

5 **(d)**

$\because \vec{\mathbf{x}} \cdot \vec{\mathbf{a}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{c}} = 0$

For non-zero vector $\vec{\mathbf{x}}$

$[\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}] = 0$ (three vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar)

and $[\vec{\mathbf{a}} \times \vec{\mathbf{b}}\ \vec{\mathbf{b}} \times \vec{\mathbf{c}}\ \vec{\mathbf{c}} \times \vec{\mathbf{a}}]$

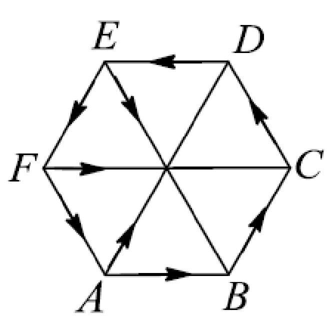
$= [\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}]^2 = 0$

6 **(d)**

$ABCDEF$ is a regular hexagon. We know from the hexagon that $\overrightarrow{\mathbf{AD}}$ is parallel to $\overrightarrow{\mathbf{BC}}$.

$\Rightarrow \overrightarrow{\mathbf{AD}} = 2\overrightarrow{\mathbf{BC}}$

Similarly, $\overrightarrow{\mathbf{EB}}$ is a parallel to $\overrightarrow{\mathbf{FA}}$



$\Rightarrow \overrightarrow{\mathbf{EB}} = 2\overrightarrow{\mathbf{FA}}$

and $\overrightarrow{\mathbf{FC}}$ is parallel to $\overrightarrow{\mathbf{AB}}$.

$\Rightarrow \overrightarrow{\mathbf{FC}} = 2\overrightarrow{\mathbf{AB}}$

Thus, $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{EB}} + \overrightarrow{\mathbf{FC}} = 2\overrightarrow{\mathbf{BC}} + 2\overrightarrow{\mathbf{FA}} + 2\overrightarrow{\mathbf{AB}}$

$= 2(\overrightarrow{\mathbf{FA}} + \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}})$

$= 2(\overrightarrow{\mathbf{FC}}) = 2(2\overrightarrow{\mathbf{AB}}) = 4\,\overrightarrow{\mathbf{AB}}$

7 **(d)**

Here, $\overrightarrow{\mathbf{a_1}} = 6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \overrightarrow{\mathbf{a_2}} = -4\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \hat{\mathbf{k}}$,

$\overrightarrow{\mathbf{b_1}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{b_2}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

∴Shortest distance

$$= \left|\frac{(\overrightarrow{\mathbf{a_2}} - \overrightarrow{\mathbf{a_1}}) \cdot (\overrightarrow{\mathbf{b_1}} \times \overrightarrow{\mathbf{b_2}})}{|\overrightarrow{\mathbf{b_1}} \times \overrightarrow{\mathbf{b_2}}|}\right|$$

$$= \left|\frac{(-10\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \cdot (8\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{\sqrt{64 + 64 + 16}}\right|$$

$$= \left|-\frac{108}{12}\right| = 9$$

8 **(c)**

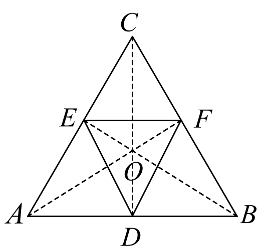
$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = 15\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 30\hat{\mathbf{k}}$$

and $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{15^2 + (-10)^2 + (30)^2} = 35$

$$\therefore \text{Required vector} = \frac{3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{7}$$

10 **(a)**

Let O be the origin



$\therefore \overrightarrow{\mathbf{BE}} + \overrightarrow{\mathbf{AF}} = \overrightarrow{\mathbf{OE}} - \overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OF}} - \overrightarrow{\mathbf{OA}}$

$$= \frac{\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OC}}}{2} - \overrightarrow{\mathbf{OB}} + \frac{\overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OC}}}{2} - \overrightarrow{\mathbf{OA}}$$

$$= \frac{\overrightarrow{\mathbf{OC}}}{2} + \frac{\overrightarrow{\mathbf{OC}}}{2} + \frac{\overrightarrow{\mathbf{OA}}}{2} - \overrightarrow{\mathbf{OA}} + \frac{\overrightarrow{\mathbf{OB}}}{2} - \overrightarrow{\mathbf{OB}}$$

$= \overrightarrow{OC} - \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2} = \overrightarrow{OC} - \overrightarrow{OD} = \overrightarrow{DC}$

11 **(d)**

$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\,|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 1 + 1 - 2\cos 60° = 2 - 1$

$\Rightarrow |\vec{a} - \vec{b}| = 1$

12 **(b)**

Given, $2\vec{a} + 3\vec{b} + \vec{c} = \vec{0}$

$\Longrightarrow 2\vec{a} + 3\vec{b} = -\vec{c}$

Taking cross product with $\vec{a}$ and $\vec{b}$ respectively, we get

$2(\vec{a} \times \vec{a}) + 3(\vec{a} \times \vec{b}) = -\vec{a} \times \vec{c}$

$\Longrightarrow 3(\vec{a} \times \vec{b}) = -\vec{c} \times \vec{a}$ ...(i)

and $2(\vec{b} \times \vec{a}) + 3(\vec{b} \times \vec{b}) = -\vec{b} \times \vec{c}$

$\Longrightarrow 2(\vec{a} \times \vec{b}) = \vec{b} \times \vec{c}$ ....(ii)

Now, $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

$= \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + 3(\vec{a} \times \vec{b})$ [using Eq. (i)]

$= 4(\vec{a} \times \vec{b}) + \vec{b} \times \vec{c}$

$= 2(\vec{b} \times \vec{c}) + \vec{b} \times \vec{c}$ [using Eq. (ii)]

$= 3(\vec{b} \times \vec{c})$

13 **(d)**

$[\vec{a} - 2\vec{b}, \vec{b} - 3\vec{c}, \vec{c} - 4\vec{a}]$

$= (\vec{a} - 2\vec{b}) \cdot \{(\vec{b} - 3\vec{c}) \times (\vec{c} - 4\vec{a})\}$

$= (\vec{a} - 2\vec{b}) \cdot \{\vec{b} \times \vec{c} - 4\vec{b} \times \vec{a} + 12\vec{c} \times \vec{a}\}$

$= (\vec{a} - 2\vec{b}) \cdot (\vec{a} + 4\vec{c} + 12\vec{b})$

$= \vec{a} \cdot \vec{a} - 24\,\vec{b} \cdot \vec{b}$

$= 1 - 24 \times 9 = 1 - 216 = -215$

14 **(b)**

Given , area $= |\vec{a} \times \vec{b}| = 15$

If the sides are $(3\vec{a} + 2\vec{b})$ and $(\vec{a} + 3\vec{b})$, then

Area of parallelogram

$= |(3\vec{a} + 2\vec{b}) \times (\vec{a} + 3\vec{b})| = 7|\vec{a} \times \vec{b}|$

$= 7 \times 15 = 105$ sq units

18 **(a)**

Given, $\vec{a} \cdot (\vec{b} + \vec{c}) = 0 \Longrightarrow \vec{a} \cdot \vec{b} + \vec{c} \cdot \vec{a} = 0$

$\vec{b} \cdot (\vec{c} + \vec{a}) = 0$

$\Longrightarrow \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{b} = 0$

and $\vec{c} \cdot (\vec{a} + \vec{b}) = 0$

$\Longrightarrow \vec{c} \cdot \vec{a} + \vec{b} \cdot \vec{c} = 0$

$\therefore\ \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$

Now, $|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|^2=|\vec{\mathbf{a}}|^2+|\vec{\mathbf{b}}|^2+|\vec{\mathbf{c}}|^2+2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})$

$\Longrightarrow|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|^2=9+16+25+0=50$

$\Longrightarrow|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|=5\sqrt{2}$

19 **(b)**

We have,

$(\vec{b}\times\vec{c})\times\vec{a}=-\{\vec{a}\times(\vec{b}\times\vec{c})\}$

$\Rightarrow(\vec{b}\times\vec{c})\times\vec{a}=-\{(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}\}=(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	D	D	D	D	D	C	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	B	D	B	C	A	D	A	B	B