DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-VECTOR ALGEBRA

1 **(d)**

We have,

$|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$

Let θ be the angle between $\vec{a}$ and $\vec{b}$

Now, $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow |\vec{c}|^2 = |\vec{a} + \vec{b}|$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 = 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 49 = 9 + 25 + 2 \times 3 \times 5 \cos\theta$

$\Rightarrow 15 = 30\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \pi/3$

2 **(c)**

$\therefore [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = \vec{\mathbf{a}} \cdot \left(|\vec{\mathbf{b}}||\vec{\mathbf{c}}| \sin\frac{2\pi}{3}\hat{\mathbf{n}}\right)$

$= |\vec{\mathbf{a}}||\vec{\mathbf{b}}||\vec{\mathbf{c}}|\left(\sin\frac{2\pi}{3}\right)$

$[\because \vec{\mathbf{a}} \cdot \hat{\mathbf{n}} = |\vec{\mathbf{a}}|\hat{\mathbf{n}}\,|\cos 0° = |\vec{\mathbf{a}}|]$

$= 2 \times 3 \times 4 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$

3 **(a)**

Given that, $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OC}} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}} - \overrightarrow{\mathbf{OA}}$

$= (3-2)\hat{\mathbf{i}} + (-2-1)\hat{\mathbf{j}} + (1+1)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{1^2 + (-3)^2 + 2^2}$

$= \sqrt{1 + 9 + 4} = \sqrt{14}$

$\overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{OC}} - \overrightarrow{\mathbf{OB}}$

$= (1-3)\hat{\mathbf{i}} + (4+2)\hat{\mathbf{j}} + (-3-1)\hat{\mathbf{k}}$

$= -2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{BC}}| = \sqrt{(-2)^2 + 6^2 + (-4)^2}$

$= \sqrt{4 + 36 + 16} = \sqrt{56}$

$\overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{OA}} - \overrightarrow{\mathbf{OC}}$

$= (2-1)\hat{\mathbf{i}} + (1-4)\hat{\mathbf{j}} + (-1+3)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{CA}}| = \sqrt{1^2 + (-3)^2 + (2)^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

It is clear that two sides of a triangle are equal.

$\therefore$ Points A,B,C from an isosceles triangle.

4 **(b)**

The component of $\vec{a}$ along $\vec{b}$ is given by

$\left\{\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right\} = \frac{18}{25}(3\hat{j} + 4\hat{k})$

5 **(a)**

It is given that $\vec{c}$ and $\vec{d}$ are collinear vectors

$\therefore \vec{c} = \lambda\,\vec{d}$ for some scalar λ

$\Rightarrow (x-2)\vec{a} + \vec{b} = \lambda\{(2x+1)\vec{a} - \vec{b}\}$

$\Rightarrow \{x - 2 - \lambda(2x+1)\}\vec{a} + (\lambda+1)\vec{b} = \vec{0}$

$\Rightarrow \lambda + 1 = 0$ and $x - 2 - \lambda(2x+1) = 0$ [$\because \vec{a}, \vec{b}$ are non-collinear]

$\Rightarrow \lambda = -1$ and $x = \frac{1}{3}$

6 **(a)**

Equation of plane is $\vec{\mathbf{r}}\cdot\hat{\mathbf{n}} = d$,

where d is the perpendicular distance of the plane from origin

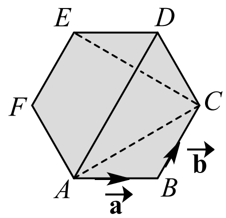
$\therefore$ Required plane is $(\alpha x + \beta y + \gamma z) = p$

7 **(c)**

In $\Delta A\,BC$, $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{AC}}$

$\Longrightarrow \overrightarrow{\mathbf{AC}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

AD is parallel to BC and $AD = 2\,BC$



$\therefore$ $\overrightarrow{\mathbf{AD}} = 2\vec{\mathbf{b}}$

In ΔACD, $\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CD}} = \overrightarrow{\mathbf{AD}}$

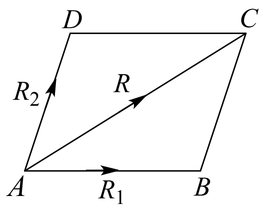
$\Longrightarrow \overrightarrow{\mathbf{CD}} = 2\vec{\mathbf{b}} - (\vec{\mathbf{a}} + \vec{\mathbf{b}}) = \vec{\mathbf{b}} - \vec{\mathbf{a}}$

Now, $\overrightarrow{\mathbf{CE}} = \overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{DE}} = \vec{\mathbf{b}} - 2\vec{\mathbf{a}}$

9 **(d)**

Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$



$\therefore$ $\vec{\mathbf{R}}$ (along $\overrightarrow{\mathbf{AC}}$) $= \vec{\mathbf{R}}_1 + \vec{\mathbf{R}}_2 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore$ $\vec{\mathbf{a}}$ (unit vector angle $\overrightarrow{\mathrm{AC}}$) $= \dfrac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|} = \dfrac{3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{9+36+4}}$

$= \dfrac{1}{7}\left(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right)$

11 **(b)**

Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors. Therefore, $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors

$\therefore x\vec{a} + y\,\vec{b} + z\,\vec{c} = \vec{0} \Rightarrow x = y = z = 0$

12 **(a)**

Suppose point $\hat{i} + 2\hat{j} + 3\hat{k}$ divides the join of points $-2\hat{i} + 3\hat{j} + 5\hat{k}$ and $7\hat{i} - \hat{k}$ in the ratio $\lambda : 1$. Then,

$\hat{i} + 2\hat{j} + 3\hat{k} = \dfrac{\lambda(7\hat{i} - \hat{k}) + (-2\hat{i} + 3\hat{j} + 5\hat{k})}{\lambda + 1}$

$\Rightarrow (\lambda+1)\hat{i} + 2(\lambda+1)\hat{j} + 3(\lambda+1)\hat{k} = (7\lambda - 2)\hat{i} + 3\hat{j} + (-\lambda + 5)\hat{k}$

$\Rightarrow \lambda + 1 = 7\lambda - 2$, $2(\lambda+1) = 3$ and $3(\lambda+1) = -\lambda + 5$

$\Rightarrow \lambda = \dfrac{1}{2}$

Hence, required ratio is 1 : 2

13 **(d)**

Clearly,

$\vec{a} - \vec{b} + \vec{b} - \vec{c} + \vec{c} - \vec{a} = \vec{0}$

$\therefore \vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}$ are coplanar

$\Rightarrow (\vec{a} - \vec{b}) \cdot \{(\vec{b} \cdot \vec{c}) \times (\vec{c} - \vec{a})\} = 0$

314 **(d)**

Two given lines intersect, if

$7\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}} + s(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$= 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}} + t(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$\Rightarrow (7 + 2s)\hat{\mathbf{i}} + (10 + 3s)\hat{\mathbf{j}} + (13 + 4s)\hat{\mathbf{k}}$

$= (3 + t)\hat{\mathbf{i}} + (5 + 2t)\hat{\mathbf{j}} + (7 + 3t)\hat{\mathbf{k}}$

$\Rightarrow 7 + 2s = 3 + t$

$\Rightarrow 2s - t = -4$ ...(i)

$10 + 3s = 5 + 2t$

$\Rightarrow 3s - 2t = -5$ ...(ii)

and 13 + 4s = 7 + 3t

$\Rightarrow 4s - 3t = -6$ ...(iii)

On solving Eqs. (i) and (iii), we get

$s = -3, t = -2$

$\therefore$ Required line is

$7\hat{\mathbf{i}}+10\hat{\mathbf{j}}+13\hat{\mathbf{k}}+(-3)[2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}]$

$\Rightarrow \hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ is the required line.

16 **(c)**

Given that, $\vec{\mathbf{a}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\vec{\mathbf{b}}=2\hat{\mathbf{i}}-\hat{\mathbf{k}}$

Let $\vec{\mathbf{r}}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$, then

$\vec{\mathbf{r}}\times\vec{\mathbf{a}}=\vec{\mathbf{b}}\times\vec{\mathbf{a}}\Rightarrow(\vec{\mathbf{r}}-\vec{\mathbf{b}})\times\vec{\mathbf{a}}=\vec{\mathbf{0}}$

Now, $\vec{\mathbf{r}}-\vec{\mathbf{b}}=(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})-(2\hat{\mathbf{i}}-\hat{\mathbf{k}})$

$=(x-2)\hat{\mathbf{i}}+y\hat{\mathbf{j}}+(z+1)\hat{\mathbf{k}}$

$$\therefore\ (\vec{\mathbf{r}}-\vec{\mathbf{b}})\times\vec{\mathbf{a}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x-2 & y & z+1\\ 1 & 1 & 0\end{vmatrix}=\vec{\mathbf{0}}$$

$\Rightarrow -(z+1)\hat{\mathbf{i}}+(z+1)\hat{\mathbf{j}}+(x-2-y)\hat{\mathbf{k}}=\vec{\mathbf{0}}$

On equating the coefficient of $\hat{\mathrm{i}},\hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$, we get

$z=-1, x-y=2$ ....(i)

Now, $\vec{\mathbf{r}}\times\vec{\mathbf{b}}=\vec{\mathbf{a}}\times\vec{\mathbf{b}}\Rightarrow(\vec{\mathbf{r}}-\vec{\mathbf{a}})\times\vec{\mathbf{b}}=\vec{\mathbf{0}}$

And $\vec{\mathbf{r}}-\vec{\mathbf{a}}=(x-1)\hat{\mathbf{i}}+(y-1)\hat{\mathbf{j}}+z\hat{\mathbf{k}}$

$$\therefore(\vec{\mathbf{r}}-\vec{\mathbf{a}})\times\vec{\mathbf{b}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x-1 & y-1 & z\\ 2 & 0 & -1\end{vmatrix}=\vec{\mathbf{0}}$$

$\Rightarrow(-y+1)\hat{\mathbf{i}}-\hat{\mathbf{j}}(-x+1-2z)+(-2y+2)\hat{\mathbf{k}}=\vec{\mathbf{0}}$

$\Rightarrow y=1, x+2z=1$ ...(ii)

From Eqs. (i) and (ii), we get

$x=3, y=1\ z=-1$

$\therefore\vec{\mathbf{r}}=\ 3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$

17 **(a)**

Given, $\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=\vec{\mathbf{B}}\times(\vec{\mathbf{C}}\times\vec{\mathbf{A}})$ ....(i)

Also, $[\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}]\neq 0\,ie.\vec{\mathbf{A}},\vec{\mathbf{B}},\vec{\mathbf{C}}$ are not coplanar.

$\therefore$ From Eq. (i)

$(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})-(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})\vec{\mathbf{C}}=(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}})\vec{\mathbf{C}}-(\vec{\mathbf{B}}\cdot\vec{\mathbf{C}})\vec{\mathbf{A}}$

$\Rightarrow(\vec{\mathbf{B}}\cdot\vec{\mathbf{C}})\vec{\mathbf{A}}+(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})\vec{\mathbf{B}}-[(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})+(\vec{\mathbf{B}}\cdot\vec{\mathbf{C}})]\vec{\mathbf{C}}=\vec{\mathbf{0}}$

$\Rightarrow\vec{\mathbf{B}}\cdot\vec{\mathbf{C}}=\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}=\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}=\vec{\mathbf{0}}$

$[\because[\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}]\neq 0]$

Now, consider

$\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})\vec{\mathbf{B}}-(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})\vec{\mathbf{C}}$

$=0\cdot\vec{\mathbf{B}}-0\cdot\vec{\mathbf{C}}=\vec{\mathbf{0}}$

319 **(a)**

$$[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$$

Applying $C_3\rightarrow C_3+C_1$

$$=\begin{vmatrix}1 & 0 & 0\\ x & 1 & 1\\ y & x & 1+x\end{vmatrix}=1[1+x-x]=1$$

Hence, $[\vec{\mathbf{a}}\ \ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$ does not depend upon neither x nor y.

20 **(b)**

The required vector is given by

$$\hat{n}=\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|}=\frac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	B	A	A	C	C	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	D	A	C	A	C	A	B