

Topic :- RELATIONS AND FUNCTIONS

1. Let $f: (-1, 1) \rightarrow B$, be a function defined by $f(x) = \tan^{-1} \frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

- a) $(-\frac{\pi}{2}, \frac{\pi}{2})$ b) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ c) $[0, \frac{\pi}{2})$ d) $(0, \frac{\pi}{2})$

2. If $f: R \rightarrow R$ defined by $f(x) = x^3$, then $f^{-1}(8)$ is equal to

- a) $\{2\}$ b) $\{2, \omega, 2\omega^2\}$ c) $\{2, -2\}$ d) $\{2, 2\}$

3. The set of all x for which there are no functions

$f(x) = \log_{(x-2)/(x+3)} 2$ and $g(x) = \frac{1}{\sqrt{x^2-9}}$, is

- a) $[-3, 2]$ b) $[-3, 2)$ c) $(-3, 2]$ d) $(-3, -2)$

4. Which of the following functions is (are) not an injective map(s)?

- a) $f(x) = |x + 1|, x \in [-1, \infty)$
 b) $g(x) = x + \frac{1}{x}, x \in (0, \infty)$
 c) $h(x) = x^2 + 4x - 5, x \in (0, \infty)$
 d) $k(x) = e^{-x}, x \in [0, \infty)$

5. If $f: N \rightarrow Z$ is defined by

$$f(n) = \begin{cases} 2 & \text{if } n = 3k, k \in Z \\ 10 & \text{if } n = 3k + 1, k \in Z \\ 0 & \text{if } n = 3k + 2, k \in Z \end{cases}$$

Then $\{n \in N: f(n) > 2\}$ is equal to

- a) $\{3, 6, 4\}$ b) $\{1, 4, 7\}$ c) $\{4, 7\}$ d) $\{7\}$

6. If $f(x) = \frac{2x-1}{x+5} (x \neq -5)$, then $f^{-1}(x)$ is equal to

- a) $\frac{x+5}{2x-1}, x \neq \frac{1}{2}$ b) $\frac{5x+1}{2-x}, x \neq 2$ c) $\frac{x-5}{2x+1}, x \neq \frac{1}{2}$ d) $\frac{5x-1}{2-x}, x \neq 2$

7. If a, b are two fixed positive integers such that

$$f(a+x) = b + [b^3 + 1 - 3b^2f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3]^{1/3}$$

For all $x \in R$, then $f(x)$ is a periodic function with period

- a) a b) $2a$ c) b d) $2b$

8. Let A be a set containing 10 distinct elements, then the total number of distinct function from A to A is

- a) 10^{10} b) 101 c) 2^{10} d) $2^{10} - 1$

9. If \mathcal{Q} denotes the set of all rational numbers and $f\left(\frac{p}{q}\right) = \sqrt{p^2 - q^2}$ for any $\frac{p}{q} \in \mathcal{Q}$, then observe the following statements.

I. $f\left(\frac{p}{q}\right)$ is real for each $\frac{p}{q} \in \mathcal{Q}$.

II. $f\left(\frac{p}{q}\right)$ is a complex number for each $\frac{p}{q} \in \mathcal{Q}$.

Which of the following is correct?

- a) Both I and II are true b) I is true, II is false
c) I is false, II is true d) Both I and II are false

10. The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

- a) $(-3, -1) \cup (1, \infty)$ b) $[-3, -1] \cup [1, \infty)$
c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ d) $[-3, -2) \cup (-2, -1) \cup (1, \infty)$

11. Let $A = \mathbb{R} - \{3\}, B = \mathbb{R} - \{1\}$. Let $f:A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$. Then,

- a) f is bijective b) f is one-one but not onto
c) f is onto but not one-one d) None of the above

12. Let $f(x) = \frac{\sqrt{\sin x}}{1 + \sqrt[3]{\sin x}}$. If D is the domain of f , then D contains

- a) $(0, \pi)$ b) $(-2\pi, -\pi)$ c) $(3\pi, 4\pi)$ d) $(4\pi, 6\pi)$

13. Let $f:\mathbb{R} \rightarrow \mathbb{R}$ and $g:\mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = 3x^2 + 2$ and $g(x) = 3x - 1$ for all $x \in \mathbb{R}$. Then,

- a) $f \circ g(x) = 27x^2 - 18x + 5$
b) $f \circ g(x) = 27x^2 + 18x - 5$
c) $g \circ f(x) = 9x^2 - 5$
d) $g \circ f(x) = 9x^2 + 15$

14. The domain of definition of the function

$$f(x) = \frac{1}{\sqrt{|x| - x}}, \text{ is}$$

- a) \mathbb{R} b) $(0, \infty)$ c) $(-\infty, 0)$ d) None of these

15. Let $f:A \rightarrow B$ and $g:B \rightarrow A$ be two functions such that $f \circ g = I_B$. Then,

- a) f and g both are injections
b) f and g both are surjections
c) f is an injection and g is a surjection
d) f is a surjection and g is an injection

16. If $f(x) = x^2 - 1$ and $g(x) = (x + 1)^2$, then $(g \circ f)(x)$ is

- a) $(x + 1)^4 - 1$ b) $x^4 - 1$ c) x^4 d) $(x + 1)^4$

17. If $f:R \rightarrow R$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^n f(r)$ is
- a) $\frac{7n}{2}$ b) $\frac{7(n+1)}{2}$ c) $7n(n+1)$ d) $\frac{7n(n+1)}{2}$
18. If $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $x^2 - 3x + 2$, then (a, b) is equal to
- a) $(-9, -2)$ b) $(6, 4)$ c) $(9, 2)$ d) $(2, 9)$
19. Let $f:R \rightarrow R$ be a function defined by $f(x) = \frac{x^2 - 8}{x^2 + 2}$. Then, f is
- a) One-one but not onto
b) One-one and onto
c) Onto but not one-one
d) Neither one-one nor onto
20. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$, is
- a) $[1, 2)$ b) $[2, 3)$ c) $[1, 2]$ d) $[2, 3]$

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