Topic :-RELATIONS AND FUNCTIONS

1   **(c)**

The period of the function in option (a) is 2. The period of the function in option (b) is 24.
The period of the function in option (c) is 2π.

2   **(a)**

We have,

$f(x) = \sqrt{3}\sin x + \cos x + 4$

$\Rightarrow f(x) = 2(\sin x \cos \pi/6 + \cos x \sin \pi/6) + 4$

$\Rightarrow f(x) = 2\sin(x + \pi/6) + 4$

Clearly, $f(x)$ will be a bijection, if $\sin(x + \pi/6)$ is a bijection

Now,

$\sin(x + \pi/6)$ is a bijection

$\Rightarrow -\pi/2 \le x + \pi/6 \le \pi/2$

$\Rightarrow -2\pi/3 \le x \le \pi/3$

$\Rightarrow x \in [-2\pi/3, \pi/3]$

For $x \in [-2/3\pi, \pi/3]$, we have

$-1 \le \sin(x + \pi/6) \le 1$

$\Rightarrow -2 \le 2\sin(x + \pi/6) \le 2$

$\Rightarrow -2 + 4 \le 2\sin(x + \pi/6) + 4 \le 2 + 4$

$\Rightarrow 2 \le f(x) \le 6$

$\Rightarrow$ Range of $f(x) = [2, 6]$

Hence, $A = [-2\pi/3, \pi/3]$ and $B = [2, 6]$

3   **(c)**

We have,

$f(x) = 2x + 3$ and $g(x) = x^2 + 7$

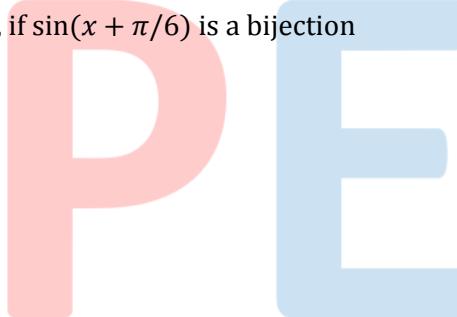$\therefore g(f(x)) = g(2x + 3) = (2x + 3)^2 + 7$

Now,

$g(f(x)) = 8$

$\Rightarrow (2x + 3)^2 + 7 = 8$

$\Rightarrow (2x + 3)^2 = 1$

$\Rightarrow 2x + 3 = \pm 1 \Rightarrow 2x = -4, -2 \Rightarrow x = -1, -2$

4   **(c)**

We have,

$$f(x) = \sin^{-1}\left(\frac{x-3}{2}\right) - \log(4-x) = g(x) + h(x)$$

where $g(x) = \sin^{-1}\left(\frac{x-3}{2}\right)$ and $h(x) = -\log(4-x)$

now, $g(x)$ is defined for

$$-1 \leq \frac{x-3}{2} \leq 1 \Rightarrow -2 \leq x-3 \leq 2 \Rightarrow 1 \leq x \leq 5$$

and, $h(x)$ is defined for $4-x > 0 \Rightarrow x < 4$

So, domain of $f(x) = [1, 5] \cap [-\infty, 4) = [1, 4)$

5 (a)

Let $y = f(x) = \frac{1-x}{1+x}$ $\qquad [\because x \neq -1]$

$$\Rightarrow \quad x = \frac{1-y}{1+y}$$

$$\therefore \quad f^{-1}(x) = \frac{1-x}{1+x} = f(x)$$

6 (b)

Since, $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ ...(i)

Replacing x by $\frac{x+59}{x-1}$ in Eq. (i), we get

$$\therefore \quad 3\left(\frac{x+59}{x-1}\right) + 2f(x) = \frac{40x+560}{x-1}$$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$f(x) = \frac{6x^2 - 4x - 242}{x-1}$$

$$\therefore \qquad f(7) = \frac{6 \times 49 - 28 - 242}{6} = 4$$

7 (c)

$$\left[\frac{2}{3} + \frac{r}{99}\right] = \begin{cases} 0, & r < 33 \\ 1, & r \geq 33 \end{cases}$$

$$\therefore \quad \sum_{r=0}^{98}\left[\frac{2}{3} + \frac{r}{99}\right] = \sum_{r=0}^{32}\left[\frac{2}{3} + \frac{r}{99}\right] + \sum_{r=33}^{98}\left[\frac{2}{3} + \frac{r}{99}\right]$$

$$= 0 + 66 = 66$$

8 (b)

We have, Domain $(f) = [0, 1]$

$\therefore f(3x^2)$ is defined, if

$0 \leq 3x^2 \leq 1$

$$\Rightarrow 0 \leq x^2 \leq \frac{1}{3} \Rightarrow |x| \leq \frac{1}{\sqrt{3}} \Rightarrow x \in [-1/\sqrt{3}, 1/\sqrt{3}]$$

9 (d)

$$\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right)$$

Since, $-2 \leq 2\sin\left(x - \frac{\pi}{3}\right) \leq 2$

$$\Rightarrow \quad -1 \leq 1 + 2\sin\left(x - \frac{\pi}{3}\right) \leq 3$$

∴ Range of $S = [-1, 3]$

10   (b)

Given,

$$f(x) = e^x \quad \text{and} \quad g(x) = \log_e x$$

Now, $f\{g(x)\} = e^{\log_e x} = x$

And $g\{f(x)\} = \log_e e^x = x$

∴ $f\{g(x)\} = g\{f(x)\}$

11   (a)

The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:

(i) $7 - x \geq 0$ (ii) $x - 3 \geq 0$ (iii) $7 - x \geq x - 3$

Now,

$$\left.\begin{array}{c} 7 - x \geq 0 \Rightarrow x \leq 7 \\ x - 3 \geq 0 \Rightarrow x \geq 3 \\ 7 - x \geq x - 3 \Rightarrow x \leq 5 \end{array}\right\} \Rightarrow 3 \leq x \leq 5$$

Hence, the required domain is $\{3, 4, 5\}$

Now,

$f(3) = {}^{7-3}P_0, f(4) = {}^3P_1 = 3$ and $f(5) = {}^2P_2 = 2$

Hence, range of $f = \{1, 2, 3\}$

12   (c)   We have,

$$f(x) = \log_{1.7}\left\{\frac{2 - \varphi'(x)}{x + 1}\right\}, \text{ where } \varphi(x) = \frac{x^3}{3} - \frac{3}{2}x^2 - 2x + \frac{3}{2}$$

For $f(x)$ to be defined, we must have

$$\frac{2 - \varphi'(x)}{x + 1} > 0, x \neq -1$$

$$\Rightarrow \frac{2 - (x^2 - 3x - 2)}{3x + 1} > 0, x \neq -1$$

$$\Rightarrow \frac{x^2 - 3x - 4}{x + 1} < 0, x \neq -1$$

$$\Rightarrow \frac{(x - 4)(x + 1)}{x + 1} < 0, x \neq -1$$

$$\Rightarrow x - 4 < 0, x \neq -1$$

$$\Rightarrow x < 4, x \neq -1$$

$$\Rightarrow x \in (-\infty, 4), x \neq -1 \Rightarrow x \in (-\infty, -1) \cup (-1, 4)$$

13   (a) $f(x)$ is defined, if

$$-1 \leq \frac{4}{3 + 2\cos x} \leq 1$$

$$\Rightarrow \frac{4}{3 + 2\cos x} \leq 1 \qquad [\because 3 + 2\cos x > 0]$$

$$\Rightarrow 4 \leq 3 + 2\cos x$$

$$\Rightarrow \cos x \geq \frac{1}{2} \Rightarrow 2n\pi - \frac{\pi}{6} \leq x \leq \frac{\pi}{6}, n \in Z$$

14   (c)

The period of the function in (a) is 2. The period of the function in (b) is 24. The period of the function in (c) is 2π

15   **(a)**

$R = \{(a, b): 1 + ab > 0\}$

It is clear that the given relation on S is reflexive, symmetric but not transitive.

17   **(a)**

We have,

$f(x) = \max\{(1 - x), 2, (1 + x)\}$

For $x \le -1$, we find that

$1 - x \ge 2$, and $1 - x \ge 1 + x$

$\therefore$ Max$\{(1 - x), 2, (1 + x)\} = 1 - x$

For $-1 < x < 1$, we find that

$0 < 1 - x < 2$, and $0 < 1 + x < 2$

$\therefore$ Max$\{(1 - x), (1 + x)\} = 2$

For $x \ge 1$, we observe that

$1 + x \ge 2$, $1 + x > 1 - x$

$\therefore$ Max$\{(1 - x), 2, (1 + x)\} = 1 + x$

Hence, $f(x) = \begin{cases} 1 - x, & x \le -1 \\ 2, & -1 < x < 1 \\ 1 + x, & x \ge 1 \end{cases}$

NOTE *Students are advised to solve this problem by drawing the graphs of $y = 1 - x$, $y = 2$ and*

$$y = 1 + x$$

18   **(d)**

Period of $\sin\frac{\theta}{3} = 6\pi$

And period of $\cos\frac{\theta}{2} = 4\pi$

$\therefore$ Period of $f(x) = \text{LCM}(6\pi, 4\pi) = 12\pi$

19   **(b)**

To make $f(x)$ an odd function in the interval $[-1, 1]$, we re-define $f(x)$ as follows:

$f(x) = \begin{cases} f(x), & 0 \le x \le 1 \\ -f(-x), & -1 \le x < 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} x^2 + x + \sin x - \cos x + \log(1 + |x|), & 0 \le x \le 1 \\ -(x^2 - x - \sin x - \cos x + \log(1 + |x|), & -1 \le x < 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} x^2 + x + \sin x - \cos x + \log(1 + |x|), & 0 \le x \le 1 \\ -x^2 + x + \sin x + \cos x - \log(1 + |x|), & -1 \le x < 0 \end{cases}$

Thus, the odd extension of $f(x)$ to the interval $[-1, 1]$ is

$-x^2 + x + \sin x + \cos x - \log(1 + |x|)$

20   **(b)**   We have,

$g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$

Now,

$f(g(x)) = 3 + 2\sqrt{x} + x$

$\Rightarrow f(g(x)) = 2 + (1 + \sqrt{x})^2$

$\Rightarrow f(g(x)) = 2 + \{g(x)\}^2$

$\Rightarrow f(x) = 2 + x^2$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	C	A	B	C	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	A	C	A	D	A	D	B	B