DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-RELATIONS AND FUNCTIONS

1 **(c)**

Given, $f(x) = x^3 - 1$

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 - 1 = x_2^3 - 1$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore f(x)$ is one-one. Also, it is onto as range of $f = R$

Hence, it is a bijection.

2 **(d)**

Given $f(x) = [x]$ and $g(x) = |x|$

Now, $f\left(g\left(\frac{8}{5}\right)\right) = f\left(\frac{8}{5}\right) = \left[\frac{8}{5}\right] = 1$

And $g\left(f\left(-\frac{8}{5}\right)\right) = g\left(\left[-\frac{8}{5}\right]\right) = g(-2) = 2$

$\therefore f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right) = 1 - 2 = -1$

3 **(a)**

$\because f(x) = \frac{\cos^{-1} x}{[x]}$

For $f(x)$ to be defined $-1 \le x \le 1$ and $[x] \ne 0 \Rightarrow x \notin [0, 1)$

$\therefore$ Domain of $f(x)$ is $[-1, 0) \cup \{1\}$.

4 **(c)**

Let $f(x) = g(x) + h(x) + u(x)$, where

$g(x) = \frac{1}{x}, h(x) = 2^{\sin^{-1} x}$ and $u(x) = \frac{1}{\sqrt{x-2}}$

The domain of $g(x)$ is the set of all real numbers other than zero i.e. $R - \{0\}$

The domain of $h(x)$ is the set $[-1, 1]$ and the domain of $u(x)$ is the set of all reals greater than 2, i.e., $(2, \infty)$

Therefore, domain of $f(x) = R - \{0\} \cap [-1, 1] \cap (2, \infty) = \phi$

5 **(b)**

Given, $2f(x) + f(1-x) = x^2$ ...(i)

Replacing x by $(1-x)$, we get

$2f(1-x) + f(x) = (1-x)^2$

$\Rightarrow 2f(1-x) + f(x) = 1 + x^2 - 2x$ ...(ii)

On multiplying Eq. (i) by 2 and subtracting from Eq. (ii), we get

$$3f(x)=x^2+2x-1 \Rightarrow f(x)=\frac{x^2-2x-1}{3}$$

6 **(d)**

$f(x)=a+bx$

$\therefore\ f\{f(x)\}=a+b(a+bx)=a(1+b)b^2x$

$\Rightarrow\ f[f\{f(x)\}]=f\{a(1+b)+b^2x\}=a(1+b+b^2)+b^3x$

$\therefore\ f^r(x)=a(1+b+b^2+\ldots+b^{r-1})+b^rx$

$$=a\left(\frac{b^r-1}{b-1}\right)+b'x$$

7 **(b)**

We have,

$f(x)=\dfrac{x-1}{x+1}$

$\Rightarrow \dfrac{f(x)+1}{f(x)-1}=\dfrac{2x}{-2}$

$\Rightarrow x=\dfrac{f(x)+1}{1-f(x)}$

$$\therefore f(2x)=\frac{2x-1}{2x+1}=\frac{2\left\{\frac{f(x)+1}{1-f(x)}\right\}-1}{2\left(\frac{f(x)+1}{1-f(x)}\right)+1}=\frac{3f(x)+1}{f(x)+3}$$

8 **(a)**

Since, $f(-x)=-f(x)$ and $f(x+2)=f(x)$

$\therefore\ f(x)=f(0)$ and $f(-2)=f(-2+2)=f(0)$

Now, $f(0)=f(-2)=-f(2)=-f(0)$

$\Rightarrow 2f(0)=0 \Rightarrow\ f(0)=0$

$\therefore\ f(4)=f(2)=f(0)=0$

9 **(c)**

We observe that $\frac{1}{x^2-36}$ is not defined for $x=\pm\ 6$

Also, $\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}$ is a real number, if

$0<\dfrac{x-1}{x++5}\le 1$

$\Rightarrow 0<\dfrac{x-1}{x+5}$ and $\dfrac{x-1}{x+5}\le 1$

$\Rightarrow (x-1)(x+5)>0$ and $1-\dfrac{6}{x+5}\le 1$

$\Rightarrow (x<-5$ or $x>1)$ and $-\dfrac{6}{x+5}\le 0$

$\Rightarrow (x<-5$ or $x>1)$ and $x+5>0$

$\Rightarrow (x<-5$ or $x>1)$ and $x>-5$

Hence, domain of $f(x)=(1,\infty)-\{6\}$

10 **(b)**

Given, $f(x) = \log_2\left(\log_3(\log_4 x)\right)$

We know, $\log_a x$ is defined, if $x > 0$

For $f(x)$ to be defined.

$\log_3\log_4 x > 0, \log_4 x > 0$ and $x > 0$

$\Rightarrow \quad \log_4 x > 3^0 = 1, x > 4^0 = 1$ and $x > 0$

$\Rightarrow \quad x > 4, x > 1$ and $x > 0$

$\Rightarrow \qquad x > 4$

11 **(c)**

We have,

$$f(x) = \begin{cases} -3x + 9, & \text{if } x < 2 \\ x - 3, & \text{if } 2 \le x < 3 \\ x - 1, & \text{if } 3 \le x < 4 \\ 3x - 9, & \text{if } x \ge 4 \end{cases}$$

$$\therefore g(x) = f(x + 1) = \begin{cases} -3x + 6, & \text{if } x < 1 \\ x - 2, & \text{if } 1 \le x < 2 \\ x, & \text{if } 2 \le x < 3 \\ 3x - 6, & \text{if } x \ge 3 \end{cases}$$

Clearly, $g(x)$ is neither even nor odd. Also, $g(x)$ is not a periodic function

12 **(b)**

We have,

$f : [2,\infty) \rightarrow B$ such that $f(x) = x^2 - 4x + 5$

Since f is a bijection. Therefore, $B =$ Range of f

Now,

$f(x) = x^2 - 4x + 5 = 5 = (x - 2)^2 + 1$ for all $x \in [2,\infty)$

$\Rightarrow f(x) \ge 1$ for all $x \in [2,\infty) \Rightarrow$ Range of $f = [1,\infty)$

Hence, $B = [1,\infty)$

13 **(d)**

Given, $R = \{(x, y): 4x + 3y = 20\}$.

Since, R is a relation on N, therefore x, y are the elements of N. But in options (a) and (b) elements are not natural numbers and option (c) does not satisfy the given relation $4x + 3y = 20$.

14 **(b)**

Since the function $f: R \rightarrow R$ given by $f(x) = x^3 + 5$ is a bijection. Therefore, f^{-1} exists

Let $f(x) = y$. Then,

$x^3 + 5 = y$

$\Rightarrow x = (y - 5)^{1/3} \qquad [\because f(x) = y \Leftrightarrow x = f^{-1}(y)]$

Hence, $f^{-1}(x) = (x - 5)^{1/3}$

15 **(a)**

We have,

$f(x) = x, g(x) = |x|$ for all $x \in R$

Now,

$[\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$

$\Rightarrow \phi(x) - f(x) = 0$ and $\phi(x) - g(x) = 0$

$\Rightarrow \phi(x) = f(x)$ and $\phi(x) = g(x)$

$\Rightarrow f(x)=g(x)=\phi(x)$
But, $f(x)=g(x)=x$, for all $x \geq 0$ [$\because |x|=x$ for all $x \geq 0$]
$\therefore \phi(x)=x$ for all $x \in [0,\infty)$
16 **(b)**
Since $f(x)$ is defined for $x \in [0, 1]$. Therefore, $f(2x+3)$ exists if
$0 \leq 2x+3 \leq 1 \Rightarrow -\frac{3}{2} \leq x \leq -1 \Rightarrow x \in [-3/2, -1]$
18 **(a)**
$fog(-1)=f\{g(-1)\}$
$\quad = f(-7) = 5-49 = -44$
19 **(a)**
We have,
$f(x)=\frac{e^{x^2}-e^{-x^2}}{e^{x^2}+e^{-x^2}}$ for all $x \in R$
Clearly, $f(-x)=f(x)$ for all $x \in R$
So, f is a many-one function
Also, $e^{x^2} > e^{-x^2} > 0$
So, $f(x)$ attains only positive values
Consequently, range of $\neq R$
Hence, f is many-one into function
20 **(c)**
Let $x, y \in N$ such that $f(x)=f(y)$
$\Rightarrow \quad x^2+x+1=y^2+y+1$
$\Rightarrow \quad (x-y)(x+y+1)=0$
$\Rightarrow \quad x=y$ or $x=(-y-1) \notin N$
$\therefore f$ one-one.
Also, f is not onto.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	A	C	B	D	B	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	A	B	B	A	A	C