DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

We observe that the periods of $\sin x$ and $\sin\frac{x}{n}$ are $\frac{2\pi}{|n|}$ and $2|n|\pi$ respectively

Therefore, $f(x)$ is periodic with period $2|n|\pi$

But, $f(x)$ has period 4π

$\therefore 2|n|\pi = 4\pi \Rightarrow |n| = 2 \Rightarrow n = \pm 2$

2 **(b)**

It can be easily checked that $f:R\to R$ given by $f(x) = \log_a(x+\sqrt{x^2+1})$ is a bijection

Now, $f\left(f^{-1}(x)\right) = x$

$\Rightarrow \log_a\left(f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2+1}\right) = x$

$\Rightarrow f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2+1} = a^x$ ...(i)

$\Rightarrow \dfrac{1}{f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2+1}} = a^{-x}$

$\Rightarrow -f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2+1} = a^{-x}$ ...(ii)

Subtracting (ii) from (i), we get

$2f^{-1}(x) = a^x - a^{-x}$

$\Rightarrow f^{-1}(x) = \frac{1}{2}(a^x - a^{-x})$

3 **(d)**

We have,

$$f(x) = x\frac{1+\frac{2}{\sqrt{x+4}}}{2-\sqrt{x+4}} + \sqrt{x+4} + 4\sqrt{x+4}$$

Clearly, $f(x)$ is defined for $x+4>0$ and $x \neq 0$

So, Domain of $f(x)$ is $(-4, 0) \cup (0,\infty)$

4 **(d)**

$\because \quad f(f(x)) = f\left(\frac{\alpha x}{x+1}\right)$

$$= \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right)+1} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$\Rightarrow \quad \frac{\alpha^2 x}{\alpha x + x + 1} = x$ [given]

$\Rightarrow \quad \alpha^2 = \alpha x + x + 1$

$\Rightarrow \quad \alpha^2 - 1 = (\alpha + 1)x$

$\Rightarrow \quad (\alpha + 1)(\alpha - 1 - x) = 0$

$\Rightarrow \quad \alpha + 1 = 0 \;\Rightarrow\; \alpha = -1 \qquad [\because \alpha - 1 - x \neq 0]$

5 **(d)**

$f(x) = \text{cosec}^2 3x + \cot 4x$

Period of $\text{cosec}^2 3x$ is $\frac{\pi}{3}$ and $\cot 4x$ is $\frac{\pi}{4}$.

$\therefore$ Period of $f(x) =$ LCM of $\left\{\frac{\pi}{3} \text{ and } \frac{\pi}{4}\right\}$

$$= \frac{\text{LCM of } (\pi, \pi)}{\text{HCF of } (3, 4)} = \frac{\pi}{1} = \pi$$

6 **(b)**

Given, $f(x) = \sqrt{1 + \log_e(1 - x)}$

For domain, $(1 - x) > 0$ and $\log_e(1 - x) \geq -1$

$\Rightarrow \quad x < 1$ and $1 - x \geq e^{-1}$

$\Rightarrow \quad x < 1$ and $x \leq 1 - \frac{1}{e}$

$\Rightarrow \quad -\infty < x \leq \frac{e - 1}{e}$

7 **(d)**

$\sin(\sin^{-1} x + \cos^{-1} x) = \sin\left(\frac{\pi}{2}\right) = 1$

$\therefore$ Range of $\sin(\sin^{-1} x + \cos^{-1} x)$ is 1.

8 **(d)**

Given, $f(x) = \cos x - \sin x$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x\right)$$

$$= \sqrt{2} \cos\left(\frac{\pi}{2} + x\right)$$

Since, $-1 \leq \cos x \leq 1 \;\Rightarrow\; -1 \leq \cos\left(\frac{\pi}{4} + x\right) \leq 1$

$\Rightarrow \quad -\sqrt{2} \leq \sqrt{2} \cos\left(\frac{\pi}{4} + x\right) \leq \sqrt{2}$

$\therefore$ Range is $[-\sqrt{2}, \sqrt{2}]$

9 **(a)**

Given, $f(x) = x^2 + \frac{1}{x^2 + 1}$

$$= (x^2 + 1) - \left(\frac{x^2}{x^2 + 1}\right)$$

$$= 1 + x^2\left(1 - \frac{1}{x^2 + 1}\right) \geq 1, \forall\, x \in R$$

Hence, range of $f(x)$ is $[1, \infty)$.

10 **(b)**

Let $y = \sqrt{\sin 2x} \Rightarrow 0 \leq \sin 2x \leq 1$,

$\Rightarrow\ 0 \leq 2x \leq \frac{\pi}{2}$

$\Rightarrow\ 0 \leq x \leq \frac{\pi}{4}$

$\Rightarrow\ x \in \left[n\pi, n\pi + \frac{\pi}{4}\right]$

11 **(c)**

We have, $f(x) = x - [x] - \frac{1}{2}$

$\therefore f(x) = \frac{1}{2} \Rightarrow x - [x] = 1$

But, for any $x \in R$, $0 \leq x - [x] < 1$

$\therefore x - [x] \neq 1$ for any $x \in R$

Hence, $\left\{x \in R : f(x) = \frac{1}{2}\right\} = \phi$

12 **(c)**

Since, $x \in [-2, 2]$, $x \leq 0$ and $f(|x|) = x$

For $-2 \leq x \leq 0$

$f(-x) = x \Rightarrow \leq (-x) - 1 = x \Rightarrow x = -\frac{1}{2}$

13 **(d)**

Given, $f(x) = \sin x$

And $g(x) = \sqrt{x^2 - 1}$

$\therefore$ Range of $f = [-1, 1] \notin$ domain of $g = (1, \infty)$

$\therefore gof$ is not defined.

14 **(d)**

Given, $f : C \to R$ such that $f(z) = |z|$

We know modulus of z and $\bar{z}$ have same values, so $f(z)$ has many one.

Also, $|z|$ is always non-negative real numbers, so it is not onto function.

15 **(b)**

We have,

$f(x) = \frac{x-1}{x+1}$

$\Rightarrow \frac{f(x)+1}{f(x)-1} = \frac{2x}{-2}$ [Applying componendo-dividendo]

$\Rightarrow x = \frac{f(x)+1}{1-f(x)}$

$$\therefore f(2x) = \frac{2x-1}{2x+1} = \frac{2\left\{\frac{f(x)+1}{1-f(x)}\right\} - 1}{2\left\{\frac{f(x)+1}{1-f(x)}\right\} + 1} = \frac{3f(x)+1}{f(x)+3}$$

16 **(b)**

Given, $f(x) = \tan\sqrt{\frac{\pi}{9} - x^2}$

For $f(x)$ to be defined $\frac{\pi^2}{9} - x^2 \geq 0$

$\Rightarrow \quad x^2 \leq \frac{\pi^2}{9} \Rightarrow -\frac{\pi}{3} \leq 3 \leq \frac{\pi}{3}$

$\therefore$ Domain of $f = \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

The greatest value of $f(x) = \tan\sqrt{\frac{\pi^2}{9}} - 0$, when $x = 0$

And the least value of $f(x) = \tan\sqrt{\frac{\pi^2}{9} - \frac{\pi^2}{9}}$, when $x = \frac{\pi}{3}$

$\therefore$ The greatest value of $f(x) = \sqrt{3}$ and the least value of $f(x) = 0$

$\therefore$ Range of $f = [0, \sqrt{3}]$.

17 **(b)**

We have,

$$[\sin x] = \begin{cases} 0, 0 \leq x < \pi/2 \\ 1, x = \pi/2 \\ 0, \pi/2 < x \leq \pi \\ -1, \pi < x < 2\pi \\ 0, x = \pi, 2\pi \end{cases}$$

And, $\operatorname{cosec}^{-1} x$ is defined for $x \in (-\infty, -] \cup [1, \infty)$

$\therefore f(x) = \operatorname{cosec}^{-1}[\sin x]$ is defined for $x = \frac{\pi}{2}$ and $x \in (\pi, 2\pi)$

Hence, domain of $\operatorname{cosec}^{-1}[\sin x]$ is $(\pi, 2\pi) \cup \left\{\frac{\pi}{2}\right\}$

18 **(a)**

aRa if $|a - a| = 0 < 1$, which is true.

$\therefore$ It is reflexive.

Now, aRb,

$|a - b| \leq 1 \Rightarrow |b - a| \leq 1$

$\Rightarrow \quad aRb \Rightarrow bRa$

$\therefore$ It is symmetric.

19 **(b)**

Given

$f(x) = \log_e(x - [x]) = \log_e\{x\}$

When x is an integer, then the function is not defined.

$\therefore$ Domain of the function $R - Z$.

20 **(b)**

Here, $f:[0, \infty] \rightarrow [0, \infty)$ *ie*, domain is $[0, \infty)$ and codomain is $[0, \infty)$.

For one-one $f(x) = \frac{x}{1+x}$

$$\Rightarrow \qquad f'(x) = \frac{1}{(1+x)^2} > 0, \forall x \in [0, \infty)$$

$\therefore f(x)$ is increasing in its domain. Thus, $f(x)$ is one-one in its domain.

For onto (we find range)

$f(x) = \frac{x}{1+x}$ *ie*, $y = \frac{x}{1+x} \Rightarrow y + yx = x$

$\Rightarrow x = \frac{y}{1-y} \Rightarrow \frac{y}{1-y} \geq 0$ as $x \geq 0 \ \therefore 0 \leq y \neq 1$

ie, Range $\neq$ Codomain

$\therefore f(x)$ is one-one but not onto.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	D	D	B	D	D	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	D	D	B	B	B	A	B	B