DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

Given, $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$ ...(i)

Replacing x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x^2}\right)+3f(x^2)=\frac{1}{x^2}-1$ ...(ii)

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and subtracting Eq. (i) from Eq. (ii), we get

$$5f(x^2)=\frac{3}{x^2}-1-2x^2$$

$$\Rightarrow \quad f(x^2)=\frac{1}{5x^2}(3-x^2-2x^4)$$

$$\Rightarrow \quad f(x^4)=\frac{1}{5x^4}(3-x^4-2x^8) \quad [\text{Replacing } x \text{ by } x^2]$$

$$=\frac{(1-x^4)(2x^4+3)}{5x^4}$$

2 **(c)**

The function $f(x)={}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:

(i)$7-x\geq 0$ (ii)$x-3\geq 0$ (iii)$7-x\geq x-3$

Now,

$$\left.\begin{aligned} 7-x\geq 0 &\Rightarrow x\leq 7\\ x-3\geq 0 &\Rightarrow x\geq 3\\ 7-x\geq x-3 &\Rightarrow x\leq 5\end{aligned}\right\}\Rightarrow 3\leq x\leq 5$$

Hence, the required domain is {3, 4, 5}

3 **(a)**

We have,

$f(x)=x$, $g(x)=|x|$ for all $x\in R$ and $\phi(x)$ satisfies the relation

$[\phi(x)-f(x)]^2+[\phi(x)-g(x)]^2=0$

$\Rightarrow \phi(x)-f(x)=0$ and $\phi(x)-g(x)=0$

$\Rightarrow \phi(x)=f(x)$ and $\phi(x)=g(x)$

$\Rightarrow f(x)=g(x)=\phi(x)$

But, $f(x)=g(x)=x$, for all $x\geq 0$ [$\because |x|=x$ for all $x\geq 0$]

$\therefore \phi(x)=x$ for all $x\in[0,\infty)$

4 **(b)**

We observe that $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ exists for

$$\frac{\pi^2}{16} - x^2 \geq 0 \Rightarrow -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

The least value of $\frac{\pi^2}{16} - x^2$ is 0 for $x = \pm\frac{\pi}{4}$ and the greatest value is $\frac{\pi^2}{16}$ for $x = 0$. Therefore, the greatest value of $f(x)$ occurs at $x = 0$ and the least value occurs at $x = \pm\pi/4$

Thus, greatest and least values of $f(x)$ are

$$f(0) = 3\sin\left(\sqrt{\frac{\pi^2}{16}}\right) = 3\sin\frac{\pi}{4} = \frac{3}{\sqrt{2}} \text{ and, } f\left(\frac{\pi}{4}\right) = 3\sin 0 = 0$$

Hence, the value of $f(x)$ lie in the interval $[0, 3/\sqrt{2}]$

ALITER For $x \in [-\pi/4, \pi/4] = Dom\ (f)$, we find that $\sqrt{\frac{\pi^2}{16} - x^2} \in [0, \pi/4]$

Since $\sin x$ is an increasing function on $[0, \pi/4]$

$$\therefore \sin x \leq \sin\sqrt{\frac{\pi^2}{16} - x^2} \leq \sin\pi/4$$

$$\Rightarrow 0 \leq 3\sin\sqrt{\frac{\pi^2}{16} - x^2} \leq \frac{3}{\sqrt{2}} \Rightarrow 0 \leq f(x) \leq \frac{3}{\sqrt{2}}$$

5 **(b)**

$$f\left(\frac{\pi}{2} + x\right) = \left|\sin\left(\frac{\pi}{2} + x\right)\right| + \left|\cos\left(\frac{\pi}{2} + x\right)\right|$$

$$= |\cos x| + |\sin x| \text{ for all } x.$$

Hence, $f(x)$ is periodic with period $\frac{\pi}{2}$.

6 **(d)**

It can be easily checked that $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$ satisfies the relation $fog\ (x) = gof\ (x)$

7 **(a)**

Since, $(1, 2) \in S$ but $(2, 1) \notin S$

$\therefore S$ is not symmetric.

Hence, S is not an equivalent relation.

Given, $T = \{(x, y) : (x - y) \in I\}$

Now, $xTx \Rightarrow x - x = 0 \in I$, it is reflexive relation

Again, $xTy \Rightarrow (x - y) \in I$

$\Rightarrow y - x \in I \Rightarrow yTx$ it is symmetric relation.

Let xTy and yTz

$\therefore x - y = I_1$ and $y - z = I_2$

Now, $x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

$\Rightarrow x - z \in I$

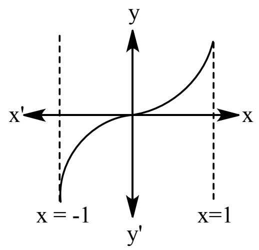
$\Rightarrow xTz$

$\therefore T$ is transitive.

Hence, T is an equivalent relation.

8 **(d)**

$$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$



Since, $-1 \leq x \leq 1$, therefore $-1 \leq f(x) \leq 1$

$\therefore$ Function is one-one onto.

9 **(c)**

We have,

$$f(x) = \frac{1-x}{1+x}$$

$$\Rightarrow f(f(x)) = f\left(\frac{1-x}{1+x}\right) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = x$$

Again,

$$f(x) = \frac{1-x}{1+x}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{x-1}{x+1}$$

$$\therefore f\left(f\left(\frac{1}{x}\right)\right) = f\left(\frac{x-1}{x+1}\right) = \frac{1 - \frac{x-1}{x+1}}{1 + \frac{x-1}{x+1}} = \frac{1}{x}$$

$$\therefore \alpha = f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right) = x + \frac{1}{x}$$

$$\Rightarrow |\alpha| = \left|x + \frac{1}{x}\right| \geq 2$$

10 **(b)**

Let $A = \{1, 2, 3\}$

Let two transitive relations on the set A are

$R = \{(1, 1), (1, 2)\}$

And $S = \{(2, 2), (2, 3)\}$

Now, $R \cup S = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$

Here, $(1, 2), (2, 3) \in R \cup S \Rightarrow (1, 3) \notin R \cup S$

$\therefore R \cup S$ is not transitive.

11 **(c)**

$f(1) = 3, f(2) = 4, f(3) = 5, f(4) = 6$

$\Rightarrow 1 \in B, 2 \in B$ do not have any pre-image in A

$\Rightarrow f$ is one-one and into

12 **(b)**

We observe that

$|f(x) + \phi(x)| = |f(x)| + |\phi(x)|$ is true, if

$f(x) \geq 0$ and $\phi(x) \geq 0$

OR

$f(x) < 0$ and $\phi(x) < 0$

$\Rightarrow (x > -1$ and $x > 2)$ or $(x < -1$ and $x < 2)$

$\Rightarrow x \in (2, \infty) \cup (-\infty, -1)$

13 **(b)**

We have, $f(x) = \dfrac{\sin^{-1}(3-x)}{\log_e(|x|-2)}$

$\sin^{-1}(3-x)$ is defined for all x satisfying

$-1 \leq 3 - x \leq 1 \Rightarrow -4 \leq -x \leq -2 \Rightarrow x \in [2, 4]$

$\log_e(|x|-2)$ is defined for all x satisfying

$|x| - 2 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

Also, $\log_e(|x|-2) = 0$ when $|x| - 2 = 1$ i.e., $x = \pm 3$

Hence, domain of $f = (2, 3) \cup (3, 4]$

14 **(a)**

$f(x)$ is defined

When $|x| > x$

$\Rightarrow \quad x < -x, x > x$

$\Rightarrow \; 2x < 0$, ($x > x$ is not possible)

$\Rightarrow \; x < 0$

Hence domain of $f(x)$ is $(-\infty, 0)$.

15 **(d)**

We have,

$f(x) = \log_{10}\{(\log_{10} x)^2 - 5(\log_{10} x) + 6\}$

Clearly, $f(x)$ assumes real values, if

$(\log_{10} x)^2 - 5\log_{10} x + 6 > 0$ and $x > 0$

$\Rightarrow (\log_{10} x - 2)(\log_{10} - 3) > 0$ and $x > 0$

$\Rightarrow (\log_{10} x < 2$ or $\log_{10} x > 3)$ and $x > 0$

$\Rightarrow (x < 10^2$ or, $x > 10^3)$ and $x > 0 \Rightarrow x \in (0, 10^2) \cup (10^3, \infty)$

16 **(b)**

We have,

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$\Rightarrow f(y) = y^2 - 2$, where $y = x + \frac{1}{x}$

Now,

$x > 0 \Rightarrow y = x + \frac{1}{x} \geq 2$ and, $x < 0 \Rightarrow y = x + \frac{1}{x} \leq -2$

Thus, $f(y) = y^2 - 2$ for all y satisfying $|y| \geq 2$

17 **(c)**

Since $\sin x$ is a periodic function with period 2π and

$$f(x) = \sin\left(\frac{2x+3}{6\pi}\right) = \sin\left(\frac{x}{3\pi} + \frac{1}{2\pi}\right)$$

$\therefore f(x)$ is periodic with period $= \frac{2\pi}{1/3\pi} = 6\pi^2$

18 **(c)**

Let $f(x) = y$. Then,

$$10x - 7 = y \Rightarrow x = \frac{y+7}{10} \Rightarrow f^{-1}(y) = \frac{y+7}{10}$$

Hence, $f^{-1}(x) = \frac{x+7}{10}$

19 **(b)**

$\therefore \ f(2.5) = [2.5 - 2] = [0.5] = 0$

20 **(c)**

We have,

$$f(x) = \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3}$$

Clearly, $f(x)$ assumes real values, if

$\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3 \geq 0$

$$\Rightarrow \log_{10}\left\{\frac{\log_{10} x}{3(4 - \log_{10} x)}\right\} \geq 0$$

$$\Rightarrow \frac{\log_{10} x}{3(4 - \log_{10} x)} \geq 1$$

$$\Rightarrow \frac{4\log_{10} x - 12}{3(4 - \log_{10} x)} \geq 0$$

$$\Rightarrow \frac{\log_{10} x - 3}{\log_{10} x - 4} \leq 0$$

$\Rightarrow 3 \leq \log_{10} x < 4 \Rightarrow 10^3 \leq x < 10^4 \Rightarrow x \in [10^3, 10^4)$

Hence, domain of $f = [10^3, 10^4)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	B	B	D	A	D	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	B	A	D	B	C	C	B	C