# DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :5

## Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

Here, $Y = \{7, 11, \dots, \infty\}$

Let $y = 4x + 3 \Rightarrow \frac{y-3}{4}$

Inverse of $f(x)$ is

$$g(y) = \frac{y-3}{4}$$

2 **(b)**

We have,

$f(x) = \sqrt{\cos(\sin x)} + \sqrt{\sin(\cos x)}$

We observe that $f(x)$ is not defined in $(\pi/2, 3\pi/2)$ and it is aperiodic function with period $2\pi$. So, let us consider the internal $[-\pi/2, \pi/2]$ as it domain. Further, since $f(x)$ is an even function. So, we will consider $f(x)$ defined on $[0, \pi/2]$ only.

Clearly, $\sqrt{\cos(\sin x)}$ and $\sqrt{\sin(\cos x)}$ are decreasing functions on $[0, \pi/2]$

Range $(f) = \left[f\left(\frac{\pi}{2}\right), f(0)\right] = [\sqrt{\cos 1}, 1 + \sqrt{\sin 1}]$

4 **(c)**

We have,

$\log x > 1$ for all $x \in (e, \infty)$

$\Rightarrow \log(\log x) > 0$ for all $x \in (e, \infty)$

$\Rightarrow f(x) - \log[\log(\log x)] \in (-\infty, \infty)$ for all $x \in (e, \infty)$

Also, $f$ is one-one. Hence, $f$ is both one-one and onto

5 **(a)**

Given, $f(x) = x^2 - 3$

Now, $f(-1) = (-1)^2 - 3 = -2$

$\Rightarrow fof(-1) = f(-2) = (-2)^2 - 3 = 1$

$\Rightarrow fofof(-1) = f(1) = 1^2 - 3 = -2$

Now, $f(0) = 0^2 - 3 = -3$

$\Rightarrow fof(0) = f(-3) = (-3)^2 - 3 = 6$

$\Rightarrow fof(0) = f(6) = 6^2 - 3 = 33$

Again, $f(1) = 1^2 - 3 = -2$

$\Rightarrow fof(1) = f(-2) = (-2)^2 - 3 = 1$

$\Rightarrow fofof(-1) + fofof(0) + fofof(1) = -2 + 33 - 2 = 29$

Now, $f(4\sqrt{2}) = (4\sqrt{2})^2 - 3 = 32 - 3 = 29$

6 **(b)**

For any $x,y \in R$, we observe that

$$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$$

So, $f$ is one-one

Let $\alpha \in R$ such that $f(x) = \alpha$

$$\Rightarrow \frac{x-m}{x-n} = \alpha \Rightarrow x = \frac{m-n\alpha}{1-\alpha}$$

Clearly, $x \in R$ for $\alpha = 1$. So, $f$ is not onto

Hence, $f$ is one-one into. This fact can also be observed from the graph of the function

7 **(b)**

We have,

$D(f) = R$ and $D(g) = R - \{0\}$

$\therefore D(h) = R - \{0\}$

Hence, $h(x) = f(x)g(x) = x \times \frac{1}{x} = 1$ for all $x \in R - \{0\}$

8 **(b)**

Since $\cos\sqrt{x}$ is not a periodic function. Therefore, $f(x) = \cos\sqrt{x} + \cos^2 x$ is not a periodic function

289 **(b)**

We have, $f(x) = 2^x$

$$\therefore \frac{f(n+1)}{f(n)} = \frac{2^{n+1}}{2^n} = 2 \text{ for all } n \in N$$

Hence, $f(0), f(1), f(2), \ldots$ are in G.P.

10 **(d)**

We have,

$f(\sin x) - f(-\sin x) = x^2 - 1$ for all $x \in R$ ...(i)

Replacing $x$ by $-x$, we get

$f(-\sin x) - f(\sin x) = x^2 - 1$ ...(ii)

Adding (i) and (ii), we get

$2(x^2 - 1) = 0 \Rightarrow x = \pm 1$

$\therefore x^2 - 2 = 1 - 2 = -1$

12 **(d)**

For $f(x)$ to be defined

$-1 \le \log_2 x \le 1$ $\quad [\because -1 \le \sin^{-1} x \le 1]$

$$\Rightarrow \frac{1}{2} \le x \le 2$$

13 **(a)**

We have,

$f(x) = |x|$ and $g(x) = [x]$

$\therefore g(f(x)) \le f(g(x))$

$\Rightarrow g(|x|) \le f([x]) \Rightarrow [|x|] \le |[x]|$

Clearly, $[|x|] = |[x]|$ for all $x \in Z$

Let $x \in (-\infty, 0)$ such that $x \notin Z$. Then, there exists positive integer $k$ such that

$-k - 1 < x < -k$

$\Rightarrow [x] = -k - 1$ and $k < |x| < k + 1$

$\Rightarrow |[x]| = k + 1$ and $[|x|] = k$

$\Rightarrow [|x|] < |[x]|$

Hence, $[|x|] \leq ||x||$ for all $x \in Z \cup (-\infty, 0)$

i.e. $\{x \in R : g(f(x)) \leq f(g(x))\} = Z \cup (-\infty, 0)$

14 **(d)**

$$\therefore f\left(\frac{3x + x^3}{1 + 3x^2}\right) - f\left(\frac{2x}{1 + x^2}\right)$$

$$= \log\left(\frac{1 + \left(\frac{3x + x^3}{1 + 3x^2}\right)}{1 - \left(\frac{3x + x^3}{1 + 3x^2}\right)}\right) - \log\left(\frac{1 + \frac{2x}{1 + x^2}}{1 - \frac{2x}{1 + x^2}}\right)$$

$$= \log\left(\frac{1 + x}{1 - x}\right)^3 - \log\left(\frac{1 + x}{1 - x}\right)^2$$

$$= \log\left(\frac{1 + x}{1 - x}\right) = f(x)$$

15 **(d)**

Clearly, $f(x)$ is defined if

$$= \underbrace{\log_{10} \log_{10} \ldots \log_{10}}_{\rightarrow (n-1) \text{ times} \leftarrow} x > 0$$

$$\Rightarrow \underbrace{\log_{10} \log_{10} \ldots \log_{10}}_{(n-2) \text{ times}} x > 1$$

$$\Rightarrow \underbrace{\log_{10} \log_{10} \ldots \log_{10}}_{(n-3) \text{ times}} x > 10$$

$$\Rightarrow x > 10^{10^{10^{\cdot^{\cdot^{(n-2) \text{ times}}}}}}$$

Thus, domain of $f = \left(10^{10^{10^{\cdot^{\cdot^{(n-2) \text{ times}}}}}}, \infty\right)$

16 **(a)**

Let $y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$

$$\Rightarrow \quad -1 \leq \log_3\left(\frac{x}{3}\right) \leq 1$$

$$\Rightarrow \quad \frac{1}{3} \leq \frac{x}{3} \leq 3$$

$$\Rightarrow \quad 1 \leq x \leq 9$$

17 **(d)**

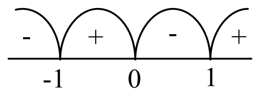
Since, $\quad f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$

For domain of $f(x)$,

$x^3 - 1 > 0,\ 4 - x^2 \neq 0)$

$\Rightarrow \ x(x - 1)(x + 1) > 0$ and $x \neq \pm 2$

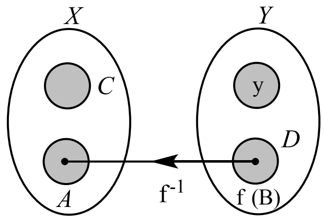
$\Rightarrow \ x \in (-1, 0) \cup (1, \infty), \quad x \neq \pm 2$




$\Rightarrow\ x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

18 **(c)**

The given data is shown in the figure below




Since, $f^{-1}(D) = x$

$\Rightarrow\ f(x) = D$

Now, if $B \subset X, f(B) \subset D$

$\Rightarrow\ f^{-1}(f(B)) = B$

19 **(b)**

Clearly, $f(x)$ is an odd function

20 **(c)**

We have,

$f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x - 1, & 0 \le x \le 2 \end{cases}$

$\therefore f(|x|) = x \quad [\because x \le 0]$

$\Rightarrow f(-x) = x$

$\Rightarrow -x - 1 = x \Rightarrow x = -\frac{1}{2}$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | B | B | C | A | B | B | B | B | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | B | D | A | D | D | A | D | C | B | C |
| | | | | | | | | | | |