DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :4

Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

Let $f^{-1}(x)=y$. Then,

$x=f(y) \Rightarrow x=3\,y-4 \Rightarrow y=\dfrac{x+4}{3}$

$\therefore f^{-1}(x)=y \Rightarrow f^{-1}(x)=\dfrac{x+4}{3}$

2 **(d)**

Here, we have to find the range of the function which is $[-1/3, 1]$

3 **(a)**

For $f(x)$ to be real, we must have

$x>0$ and $\log_{10} x \neq 0$

$\Rightarrow x>0$ and $x \neq 1 \Rightarrow x>0$ and $x \neq 1 \Rightarrow x \in (0,1) \cup (1,\infty)$

4 **(a)**

Let $W=\{cat, toy, you, \ldots\}$

Clearly, R is reflexive and symmetric but not transitive.

[Since, ${}_{cat}R_{toy}, {}_{toy}R_{you} \nRightarrow {}_{cat}R_{you}$]

5 **(c)**

Given, $f(x)=\frac{ax+b}{cx+d}$

It reduces the constant function if

$$\frac{a}{c}=\frac{b}{d} \Rightarrow ad=bc$$

7 **(c)**

Since, the relation R is defined as

$R=\{(x,y)|x, y$ are real numbers and $x=wy$ for some rational number $w\}$

(i) Reflexive $xRx \Rightarrow x=wx$

$\therefore \quad w=1 \in$ Rational number

$\Rightarrow$ The relation R is reflexive.

(ii) Symmetric $xRy \Rightarrow yRx$

As $0R1$

$\Rightarrow\ 0=0\ (1)$ but $1R0 \Rightarrow 1=w.(0)$,

Which is not true for any rational number

$\Rightarrow$ The relation R is not symmetric

Thus, R is not equivalent relation.

Now, for the relation S is defined as

$S = \left\{\left(\frac{m}{n}, \frac{m}{n}\right)\right|$

m, n, p and $q \in$ integers such that $n, q \neq 0$ and $qm = pn\}$

(i) Reflexive $\frac{m}{n} S \frac{m}{n} \Rightarrow mn = mn$ (True)

$\Rightarrow$ The relation S is reflexive

(ii) Symmetric $\frac{m}{n} S \frac{p}{q} \Rightarrow mq = np$

$\Rightarrow np = mq \Rightarrow \frac{p}{q} S \frac{m}{n}$

$\Rightarrow$ The relation S is symmetric.

(iii) Transitive $\frac{m}{n} S \frac{p}{q}$ and $\frac{p}{q} S \frac{r}{s}$

$\Rightarrow mq = np$ and $ps = rq$

$\Rightarrow mq.ps = np.rq$

$\Rightarrow ms = nr \quad \Rightarrow \frac{m}{n} = \frac{r}{s} \Rightarrow \frac{m}{n} S \frac{r}{s}$

$\Rightarrow$ The relation S is transitive

$\Rightarrow$ The relation S is equivalent relation.

8 **(a)**

We know that $\tan x$ has period π. Therefore, $|\tan x|$ has period $\frac{\pi}{2}$. Also, $\cos 2x$ has period π.

Therefore, period of $|\tan x| + \cos 2x$ is π.

Clearly, $2\sin\frac{\pi x}{3} + 3\cos\frac{2\pi x}{3}$ has its period equal to the LCM of 6 and 3 i.e., 6

$6\cos(2\pi x + \pi/4) + 5\sin(\pi x + 3\pi/4)$ has period 2

The function $|\tan 4x| + |\sin 4x|$ has period $\frac{\pi}{2}$

9 **(a)**

Let $y = f(x) = \sqrt{(x-1)(3-x)}$

$\Rightarrow \; x^2 - 4x + 3 + y^2 = 0$

This is a quadratic in x, we get

$$x = \frac{+4 \pm \sqrt{16 - 4(3 + y^2)}}{2(1)} = \frac{4 \pm 2\sqrt{1 - y^2}}{2(1)}$$

Since, x is real, then $1 - y^2 \geq 0 \Rightarrow -1 \leq y \leq 1$

But $f(x)$ attains only non-negative values.

Hence, $y = f(x) = [0, 1]$

10 **(d)**

$\{(z, b), (y, b), (a, d)\}$ is not a relation from A to B because $a \notin A$

12 **(a)**

For $x \geq 1$, we have

$x \leq x^2 \Rightarrow \min\{x, x^2\} = x$

For $0 \leq x < 1$, we have,

$x^2 < x \Rightarrow \min\{x, x^2\} = x^2$

For $x < 0$, we have

$x < x^2 \Rightarrow \min\{x, x^2\} = x$

Hence, $f(x) = \min\{x, x^2\} = \begin{cases} x, & x > 1 \\ x^2, & 0 \le x < 1 \\ x, & x < 0 \end{cases}$

<u>ALITER</u> Draw the graphs of $y = x$ and $y = x^2$ to obtain $f(x)$

13 **(a)**

Clearly, mapping f given in option (a) satisfies the given conditions

14 **(b)**

Given, $f(x) = e^{\sqrt{5x-3-2x^2}}$

For domain of $f(x)$

$$2x^2 - 5x + 3 \le 0$$

$\Rightarrow (2x-3)(x-1) \le 0$

$$\Rightarrow \quad 1 \le x \le \frac{3}{2}$$

$\therefore$ Domain of $f(x) = \left[1, \frac{3}{2}\right]$.

15 **(d)**

Given, $f(x) = x + \sqrt{x^2}$

Since, this function is not defined

16 **(a)**

We have,

$$f(x) = \frac{\sin^4 x + \cos^2 x}{\sin^2 x + \cos^4 x}$$

$$\Rightarrow f(x) = \frac{(1-\cos^2 x)^2 + \cos^2 x}{1 - \cos^2 x + \cos^4 x} = 1 \quad \text{for all } x \in R$$

$\therefore f(2010) = 1$

17 **(c)**

We have,

$$f(x) = \log\{ax^3 + (a+b)x^2 + (b+c)x + c\}$$

$$\Rightarrow f(x) = \log\{(ax^2 + bx + c)(x+1)\}$$

$$\Rightarrow f(x) = \log\left\{a\left(x + \frac{b}{2a}\right)^2 (x+1)\right\}$$

$$\Rightarrow f(x) = \log a + \log\left(x + \frac{b}{2a}\right)^2 + \log(x+1)$$

Since $a > 0$, therefore $f(x)$ is defined for $x \ne -\frac{b}{2a}$ and $x + 1 > 0$

i.e., $x \in R - \left\{\left\{-\frac{b}{2a}\right\} \cap (-\infty, -1)\right\}$

18 **(a)**

$$\because \quad y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$$

$$\Rightarrow \quad \frac{y+1}{y-1} = \frac{10^x}{-10^{-x}}$$

[using componendo and dividendo rule]

$\Rightarrow \quad 10^{2x} = \frac{1+y}{1-y}$

$\Rightarrow \quad 2x \log_{10} 10 = \log_{10}\left(\frac{1+y}{1-y}\right)$

$\Rightarrow \quad x = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$

$\therefore \quad f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$

19 **(b)**

Given, $f(x) = \begin{cases} -1, \text{ when } x \text{ is rational} \\ 1, \text{ when } x \text{ is irrational} \end{cases}$

Now, $(fof)(1-\sqrt{3}) = f[f(1-\sqrt{3})] = f(1) = -1$

20 **(c)**

We have,

$f(x) = 6^x + 6^{|x|} > 0$ for all $x \in R$

$\therefore$ Range $(f) \neq$ (Co − domain (f)

So, $f{:}R \rightarrow R$ is an into function

For any $x, y \in R$, we find that

$x \neq y \Rightarrow 2^x \neq 2^y \Rightarrow 2^{x+|x|} \neq 2^{y+|y|} \Rightarrow f(x) \neq f(y)$

So, f is one-one

Hence, f is a one-one into function

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	A	A	C	C	C	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	A	B	D	A	C	A	B	C