DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-RELATIONS AND FUNCTIONS

1 **(d)**

We observe that

Period of $\sin\frac{\pi x}{2}$ is $\frac{2\pi}{\pi/2}=4$, Period of $\cos\frac{\pi x}{3}$ is $\frac{2\pi}{\pi/3}=6$,

and,

Period of $\tan\frac{\pi x}{4}$ is $\frac{\pi}{\pi/4}=4$

$\therefore$ Period of $f(x)=$ LCM of $(4, 6, 4)=12$

2 **(c)**

We have,

$$f(x)=\lim_{x\to\infty}\frac{x^n+x^{-n}}{x^n+x^{-n}}$$

$$\Rightarrow f(x)=\lim_{x\to\infty}\frac{x^{2n}-1}{x^{2n}+1}=\frac{0-1}{0+1}=-1, \text{ if } -1<x<1$$

If $|x|>1$, then $x^{2n}\to\infty$ as $n\to\infty$

$$\therefore f(x)=\lim_{x\to\infty}\frac{1-\frac{1}{x^{2n}}}{1+\frac{1}{x^{2n}}}=\frac{1-0}{1+1},=1, \text{ if } |x|>1$$

If $|x|=1$, then $x^{2n}=1$

$$\therefore f(x)=\lim_{x\to\infty}\frac{x^{2n}-1}{x^{2n}+1}=\frac{1-1}{1+1}=0$$

Thus, we have

$$f(x)=\begin{cases}-1, & \text{if } |x|<1\\ 0, & \text{if } |x|=1\\ 1, & \text{if } |x|>1\end{cases}$$

3 **(c)**

$R=\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ is a relation on $A=\{1,2,3,4\}$, then

(a) since, $(2,4)\in R$ and $(2,3)\in R$, so R is not a function.

(b) since, $(1,3)\in R$ and $(3,1)\in R$ but $(1,1)\notin R$.

So, R is not transitive.

(c) since, $(2,3)\in R$ but $(3,2)\notin R$, so R is not symmetric.

(d) since, $(4,4)\notin R$, so R is not reflexive.

4 **(a)**

We have,

$f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$

Clearly, $f(x)$ is defined, if

$16 - x \geq 2x - 1 > 0, 20 - 3x \geq 4x - 5 > 0$ and $x \in Z$

$\Rightarrow x \in \{1,2,3,4,5\}, x \in \{2, 3\}$ and $x \in Z$

$\Rightarrow x \in \{2,3\}$

$\therefore$ Domain $(f) = \{2,3\}$

5 **(d)**

Given, $f(x) = e^{2ix}$ and $f{:}R\rightarrow C$. Function $f(x)$ is not one-one, because after some values of $x(ie, \pi)$ it will give the same values.

Also, $f(x)$ is not onto, because it has minimum and maximum values $-1 - i$ and $1 + i$ respectively.

6 **(a)**

For $f(x)$ to be defined,

$x - 4 \geq 0$ and $6 - x \geq 0 \Rightarrow x \geq 4$ and $x \leq 6$

Therefore, the domain is [4, 6].

7 **(d)**

We have,

$hogof\ (x) = \cos^{-1}(|\sin x|)$

and, $fogoh\ (x) = \sin^2\left(\sqrt{\cos^{-1} x}\right)$

Clearly, $hogof\ (x) \neq fogoh(c)$

Thus, option (a) is not correct

Now,

$gofoh\ (x) = \left|\sin\left(\cos^{-1} x\right)\right| = \left|\sin\left(\sin^{-1}\sqrt{1 - x^2}\right)\right| = \sqrt{1 - x^2}$

and, $fohog\ (x) = \sin^2(\cos^{-1}\sqrt{x}) = 1 - \cos^2(\cos^{-1}\sqrt{x})$

$\Rightarrow fohog\ (x) = 1 - \left\{\cos\left(\cos^{-1}\sqrt{x}\right)\right\}^2 = 1 - x$

$\therefore gofoh\ (x) \neq fohog(x)$

Thus, option (b) is correct

Also,

$hogof(x) = \cos^{-1}(|\sin x|)$ and, $fohog\ (x) = 1 - x$

$\therefore hogof(x) \neq fohog(x)$

Thus, option (c) is not correct

Hence, option (d) is correct

8 **(a)**

We have,

$$f(x) = \frac{2^x + 2^{-x}}{2}$$

$$\therefore f(x + y)f(x - y) = \frac{2^{x+y} + 2^{-x-y}}{2} \times \frac{2^{x-y} + 2^{-x+y}}{2}$$

$$\Rightarrow f(x + y)f(x - y) = \frac{2^{2x} + 2^{-2y} + 2^{2y} + 2^{-2x}}{4}$$

$$\Rightarrow f(x+y)f(x-y)=\frac{1}{2}\left(\frac{2^{2x}+2^{-2x}}{2}+\frac{2^{2y}+2^{-2y}}{2}\right)$$

$$\Rightarrow f(x+y)\ f\ (x-y)=\frac{1}{2}\{f(2x)+f\ (2y)\}$$

9 **(b)**

$R=\{(a,b):a,b\in N,a-b=3\}$

$=\{[(n+3),n]:n\in N\}$

$=\{(4,1),(5,2),(6,3),....\}$

10 **(a)**

Clearly, $f(x)=\sin^{-1}\left\{\log_3\left(\frac{x}{3}\right)\right\}$ exists if

$$-1\le\log_3\left(\frac{x}{3}\right)\le 1\Leftrightarrow 3^{-1}\le\frac{x}{3}\le 3^1\Leftrightarrow 1\le x\le 9$$

Hence, domain of $f(x)$ is $[1,9]$

11 **(c)**

For $f(x)$ to be defined, we must have

$$\frac{\sqrt{4-x^2}}{1-x}>0,4-x^2>0 \text{ and } 1-x\ne 0$$

$\Rightarrow 1-x>0,4-x^2>0$ and $1-x\ne 0$

$\Rightarrow x<1,x\in(-2,2)$ and $x\ne 1\Rightarrow x\in(-2,1)$

$\therefore$ Domain $(f)=(-2,1)$

Now, for $x\in(-2,1)$, we have

$$-\infty<\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)<\infty$$

$$\Rightarrow -1\le\sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}\le 1\Rightarrow -1\le f(x)\le 1$$

Hence, Range $(f)=[-1,1]$

12 **(a)**

Given, $f(x)=\frac{ax+b}{cx+d}$ and $fof(x)=x$

$$\Rightarrow\quad f\left(\frac{ax+b}{cx+d}\right)=x$$

$$\Rightarrow\quad \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d}=x$$

$$\Rightarrow\quad \frac{x(a^2+bc)+ab+bd}{x(ac+cd)+bc+d^2}=x$$

$\Rightarrow\quad d=-a$

13 **(c)**

If $f:C\to C$ given by $f(x)=\frac{ax+b}{cx+d}$ is a constant function, then

$f(x)=$ Constant $(=\lambda$, say) for all $x\in C$

$\Rightarrow \dfrac{ax+b}{cx+d} = \lambda$ for all $x \in C$

$\Rightarrow (a-\lambda c)\,x + (b-\lambda d) = 0$ for all $x \in C$

$\Rightarrow a - \lambda c = 0$ and $b - \lambda d = 0 \Rightarrow \frac{a}{c} = \frac{b}{d} \Rightarrow ad = bc$

14 **(d)**

Periods of $\sin\lambda\, x + \cos\lambda\, x$ and $|\sin x| + |\cos x|$ are $\frac{2\pi}{\lambda}$ and $\frac{\pi}{2}$ respectively

$\therefore \dfrac{\pi}{2} = \dfrac{2\pi}{\lambda} \Rightarrow \lambda = 4$

15 **(b)**

We have, $f(x) = \sqrt{\log_{16} x^2}$

Clearly, $f(x)$ exists, if

$\log_{16} x^2 \geq 0 \Rightarrow x^2 \geq 1 \Leftrightarrow |x| \geq 1$

16 **(b)**

Since, $f(x)$ is an even function, therefore $f'(x)$ is an odd function

$ie, \quad f'(-e) = -f'(e)$

$\therefore \; f'(e) + f'(-e) = 0$

17 **(c)**

We have,

$f(x) = \log\left(\dfrac{1+x}{1-x}\right)$

$$\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left\{\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right\} = \log\left(\frac{x+1}{1-x}\right)^2$$

$$\Rightarrow f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+x}{1-x}\right) = 2\,f\,(x)$$

18 **(c)**

$f(x) = \cos^2 x + \sin^4 x = 1 - \cos^2 x + \cos^4 x$

$\Rightarrow f(x) = \left(\cos^2 x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4} \geq \dfrac{3}{4}$ for all x

Also, $f(x) = \cos^2 x + \sin^4 x \leq \cos^2 x + \sin^2 x = 1$

$\therefore$ Range $(f) = [3/4, 1]$

Hence, $f(R) = [3/4, 1]$

19 **(d)**

For domain of given function

$$-1 \leq \log_2\left\{\frac{x}{12}\right\} \leq 1$$

$\Rightarrow\; 2^{-1} \leq \dfrac{x}{12} \leq 2$

$\Rightarrow\; 6 \leq x \leq 24$

$\Rightarrow\; x \in [6, 24]$

20 **(d)**

Given, $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$

Here, 4^{-x^2} is defined for $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$,$\cos^{-1}\left(\frac{x}{2} - 1\right)$ is defined,

If $-1 \leq \frac{x}{2} - 1 \leq 1 \Rightarrow 0 \leq x \leq 4$

And $\log(\cos x)$ is defined, if $\cos x > 0$

$\Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

Hence, $f(x)$ is defined for $x \in \left[0, \frac{\pi}{2}\right]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	C	A	D	A	D	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	D	B	B	C	C	D	D