

## Topic :-RELATIONS AND FUNCTIONS

1 (c)

Clearly,  $X = R^+$  and  $Y = R$

2 (b)

Given,  $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

Let  $f(x) = x^n \pm 1$ , where  $n \in I$ .

Now,  $f(4) = 65$

Case I

Let  $f(x) = x^n + 1$

$\Rightarrow f(4) = 4^n + 1$

$\Rightarrow 65 = 4^n + 1$

$\Rightarrow n = 3$

Case II

Let  $f(x) = x^n - 1$

$\Rightarrow f(4) = 4^n - 1 \Rightarrow 65 = 4^n - 1$

$\Rightarrow 4^n = 66$

The quality does not hold true for  $n \in Z$ .

Therefore,  $f(x) = x^3 + 1$

Now,  $f(6) = 6^3 + 1 = 216 + 1 = 217$

3 (b)

Since, the graph is symmetrical about the line  $x = 2$

$\Rightarrow f(2 + x) = f(2 - x)$

4 (c)

We have,

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases} \text{ and } g(x) = x(1 - x^2)$$

$\therefore fog(x) = f(g(x))$

$$\Rightarrow fog(x) = \begin{cases} -1, & \text{if } g(x) < 0 \\ 0, & \text{if } g(x) = 0 \\ 1, & \text{if } g(x) > 0 \end{cases}$$

$$\Rightarrow fog(x) = \begin{cases} -1, & \text{if } x \in (-1, 0) \cup (1, \infty) \\ 0, & \text{if } x = 0, \pm 1 \\ 1, & \text{if } x \in (-\infty, -1) \cup (0, 1) \end{cases}$$

5 (b)

**Reflexive**  $xRx$

Since,  $x^2 = x.x$

$$x^2 = xy$$

Transitive,  $xRy \Rightarrow x^2 = xy$

And  $yRz \Rightarrow y^2 = yz$

Now,  $x^2y^2 = xy^2z \Rightarrow x^2 = xz$

$\Rightarrow xRz$

$\therefore$  It is transitive.

6 **(c)**

We have,

$$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right), n \in \mathbb{Z}, n > 2$$

Since  $\sin\left(\frac{\pi x}{n-1}\right)$  and  $\cos\left(\frac{\pi x}{n}\right)$  are periodic functions with period  $2(n-1)$  and  $2n$  respectively.

Therefore,  $f(x)$  is periodic with period equal to LCM of  $(2n, 2(n-1)) = 2n(n-1)$

7 **(b)**

Let  $g(x)$  be the even extension of  $f(x)$  on  $[-4, 4]$

Then,

$$g(x) = \begin{cases} f(x) & \text{for } x \in [-4, 0] \\ f(-x) & \text{for } x \in [0, 4] \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} e^x + \sin x & \text{for } x \in [-4, 0] \\ e^{-x} + \sin(-x) & \text{for } x \in [0, 4] \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} e^x + \sin x & \text{for } x \in [-4, 0] \\ e^{-x} - \sin x & \text{for } x \in [0, 4] \end{cases}$$

$$\Rightarrow g(x) = e^{-|x|} - \sin|x| \text{ for } x \in [-4, 4]$$

8 **(d)**

Clearly,  $f(x)$  is an even function and  $f(x) < 0$  for all  $x > 0$

Therefore, the graph of  $f(x)$  lies in the third and fourth quadrants

9 **(d)**

The given function is

$$f(x) = \sqrt{1-2x} + 2 \sin^{-1}\left(\frac{3x-1}{2}\right)$$

For domain of  $f(x)$ ,  $1-2x \geq 0$  and  $-1 \leq \frac{3x-1}{2} \leq 1$

$$\Rightarrow x \leq \frac{1}{2} \text{ and } -2 \leq 3x-1 \leq 2$$

$$\Rightarrow x \leq \frac{1}{2} \text{ and } -\frac{1}{3} \leq x \leq 1$$

$$\therefore \text{Domain of } f(x) = \left[-\frac{1}{3}, \frac{1}{2}\right]$$

10 **(c)**

We have,

$$f(x) = \log_{(x+3)}(x^2-1)$$

Clearly,  $f(x)$  is defined for  $x$  satisfying the following conditions

(i)  $x^2-1 > 0$  (ii)  $x+3 > 0$  and  $x+3 \neq 1$

Now,  $x^2-1 > 0 \Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

and,

$$x + 3 > 0 \text{ and } x + 3 \neq 1 \Rightarrow x > -3 \text{ and } x \neq -2$$

$$\Rightarrow x \in (-3, -2) \cup (-2, \infty)$$

Hence, the domain of  $f(x)$  is  $(-3, -2) \cup (-2, -1) \cup (1, \infty)$

11 (b)

$$x^2 - 6x + 7 = (x - 3)^2 - 2$$

Obviously, minimum value is  $-2$  and maximum is  $\infty$ .

12 (d)

We have,

$$f \circ f^{-1}(x) = x$$

$$\Rightarrow f(f^{-1}(x)) = x$$

$$\Rightarrow f(y) = x \text{ where } y = f^{-1}(x)$$

$$\Rightarrow \frac{e^y - e^{-y}}{e^y + e^{-y}} + 2 = x \Rightarrow \frac{e^y - e^{-y}}{e^y + e^{-y}} = x - 2 \Rightarrow \frac{2e^y}{-2e^{-y}} = \frac{x - 1}{x - 3}$$

$$\Rightarrow e^{2y} = \frac{x - 1}{3 - x}$$

$$\Rightarrow y = \frac{1}{2} \log \left( \frac{x - 1}{3 - x} \right)$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2} \log \left( \frac{x - 1}{3 - x} \right)$$

13 (b)

$$f(x) = \frac{4^x}{4^x + 2}$$

$$\begin{aligned} \therefore f(1 - x) + f(x) &= \frac{4^{1-x}}{4^{1-x} + 2} + \frac{4^x}{4^x + 2} \\ &= \frac{4}{4 + 2 \cdot 4^x} + \frac{4^x}{4^x + 2} = \frac{2}{2 + 4^x} + \frac{4^x}{4^x + 2} = 1 \end{aligned}$$

By putting  $x = \frac{1}{97}, \frac{2}{97}, \frac{3}{97}, \dots, \frac{48}{97}$

And adding, we get

$$f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \dots + f\left(\frac{96}{97}\right) = 48$$

14 (c)

$$\begin{aligned} \text{Given, } f(x) &= \frac{2 \sin 8x \cos x - 2 \sin 6x \cos 3x}{2 \cos 2x \cos x - 2 \sin 3x \sin 4x} \\ &= \frac{(\sin 9x + \sin 7x) + (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) + (\cos 7x - \cos x)} \\ &= \frac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x} \\ &= \frac{2 \cos 5x \sin 2x}{2 \cos 2x \cos 5x} = \tan 2x \end{aligned}$$

$\therefore$  Period of  $f(x) = \frac{\pi}{2}$

15 (d)

$$g \circ f = g\{f(x)\} = g(x^2) = x^2 + 5$$

16 **(b)**

We have,

$$f(x) = \log_{2x-5}(x^2 - 3x - 10)$$

For  $f(x)$  to be defined, we must have

$$x^2 - 3x - 10 > 0, 2x - 5 > 0 \text{ and } 2x - 5 \neq 1$$

$$\Rightarrow (x - 5)(x + 2) > 0, x > \frac{5}{2} \text{ and } x \neq 3$$

$$\Rightarrow x > 5 \Rightarrow x \in (5, \infty)$$

17 **(c)**

Since,  $f(x)$  is an even function therefore its values is always greater than equal to 0 and we know

$$x^2 < x^2 + 1 \text{ or } \frac{x^2}{x^2 + 1} < 1$$

$\therefore$  Required range is  $[0, 1)$ .

18 **(d)**

We have,

$$f(x^2) = |x^2 - 1| \neq |x - 1|^2 = [f(x)]^2$$

$$f(|x|) = ||x| - 1| \neq |x - 1| = |f(x)|$$

And,

$$f(x + y) = |x + y - 1| \neq |x - 1| + |y - 1| = f(x) + f(y)$$

Hence, none of the above given option is true

19 **(d)**

We have,

$$\begin{aligned} f(x + 2) - 2f(x + 1) + f(x) \\ &= a^{x+2} - 2a^{x+1} + a^x \\ &= a^x(a^2 - 2a + 1) = a^x(a - 1)^2 = (a - 1)^2 f(x) \end{aligned}$$

So, option (a) holds

It can be easily checked that options (b) and (c) are also true but option (d) is not true

20 **(b)**

It can be easily seen that  $f:A \rightarrow A$  is a bijection. Let  $f(x) = y$ . Then,

$$f(x) = y$$

$$\Rightarrow x(2 - x) = y$$

$$\Rightarrow x^2 - 2x + y = 0$$

$$\Rightarrow x^2 - 2x + y = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4 - 4y}}{2}$$

$$\Rightarrow x = 1 \pm \sqrt{1 - y}$$

$$\Rightarrow x = 1 - \sqrt{1 - y} \quad [\because x \leq 1]$$

$$\Rightarrow f^{-1}(y) = 1 - \sqrt{1 - y}$$

Hence,  $f^{-1}:A \rightarrow A$  is defined as  $f^{-1}(x) = 1 - \sqrt{1 - x}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	C	B	C	B	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	B	C	D	B	C	D	D	B

PE