DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :10

Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

Given, $f(x) = \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x(x^2 < 1)$

Since, $x \in (-1, 1)$.

$\Rightarrow \tan^{-1}x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow 2\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

So, $f(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

2 **(a)**

Let $y = f(x) = x^3$

$\therefore x = y^{1/3}$

$\Rightarrow f^{-1}(x) = x^{1/3}$

$\therefore f^{-1}(8) = (8)^{1/3} = 2$

3 **(d)**

For $f(x) = \log_{\left(\frac{x-2}{x+3}\right)} 2$ to exist, we must have

$\frac{x-2}{x+3} > 0$ and $\frac{x-2}{x+3} \neq 1 \Rightarrow x < -3$ or $x > 2, x \neq -3, x \neq 2$

For $g(x) = \frac{1}{\sqrt{x^2-9}}$ to exist, we must have

$x^2 - 9 > 0 \Rightarrow x < -3$ or $x > 0$

Thus, $f(x)$ and $g(x)$ both do not exist for $-3 < x < 2$, i.e., for $x \in (-3, 2)$

4 **(b)**

For choice (a), we have

$f(x) = f(y), x,y \in [-1, \infty)$

$\Rightarrow |x+1| = |y+1| \Rightarrow x+1 = y+1 \Rightarrow x = y$

So, f is an injection

For choice (b), we have

$g(2) = \frac{5}{2}$ and $g(1/2) = \frac{5}{2}$

$\therefore 2 \neq \frac{1}{2}$ but $g(2) = g(1/2)$

Thus, $g(x)$ is not injective

It can be easily seen that choices $h(x)$ and $k(x)$ are injections

5 **(b)**

We have

$$f(n)=\begin{cases} 2 & \text{if } n=3k, \quad k\in Z \\ 10 & \text{if } n=3k+1, k\in Z \\ 0 & \text{if } n=3k+2, \quad k\in Z \end{cases}$$

For $f(n)>2$, we take $n=3k+1, k\in Z$

$\Rightarrow \quad n=1,4,7$

$\therefore$ Required set $\{n\in Z; f(n)>2\}=\{1,4,7\}$

6 **(b)**

Let $\quad y=\frac{2x-1}{x+5}$

$\Rightarrow \quad x=\dfrac{5y+1}{2-y}$

$\therefore f^{-1}(x)=\dfrac{5x+1}{2-x}, x\neq 2$

7 **(b)**

We have,

$f(a+x)=b+\left[b^3+1-3b^2f(x)+3b\{f(x)\}^2-\{f(x)\}^3\right]^{1/3}$ for all $x\in R$

$\Rightarrow f(a+x)=b+\left[1+\{b-f(x)\}^3\right]^{1/3}$ for all $x\in R$

$\Rightarrow f(a+x)-b=\left[1-\{f(x)-b\}^3\right]^{1/3}$ for all $x\in R$

$\Rightarrow g(a+x)=\left[1-\{g(x)\}^3\right]^{1/3}$ for all $x\in R$,

Where $g(x)=f(x)-1$

$\Rightarrow g(2a+x)=\left[1-\{g(a+x)\}^3\right]^{1/3}$ for all $x\in R$

$\Rightarrow g(2a+x)=\left[1-\{1-(g(x))^3\}\right]^{1/3}$ for all $x\in R$

$\Rightarrow g(2a+x)=g(x)$ for all $x\in R$

$\Rightarrow f(2a+x)-1=f(x)-1$ for all $x\in R$

$\Rightarrow f(2a+x)=f(x)$ for all $x\in R$

$\Rightarrow f(x)$ is periodic with period $2a$

8 **(a)**

Given a set containing 10 distinct elements and $f:A\to A$ Now, every element of a set A can make image in 10 ways.

$\therefore$ Total number of ways in which each element make images $=10^{10}$.

9 **(c)**

Given, $f\left(\frac{p}{q}\right)=\sqrt{p^2-q^2}$, for $\frac{p}{q}=Q$

If $p<q$, then $f\left(\frac{p}{q}\right)$ is not real.

Hence, statement I is false while statement II is true.

10 **(c)**

The given function is defined when $x^2-1; 3+x>0$ and $3+x\neq 1$

$\Rightarrow \quad x^2>1;\ 3+x>0$ and $x\neq -2$

$\Rightarrow \quad -1>x>1;\ x>-3, \quad x\neq -2$

$\therefore$ Domain of the function is

$D_f=(-3,-2)\cup(-2,-1)\cup(1,\infty)$

391 **(a)**

Let x and y be two arbitary elements in A.
Then, $f(x)=f(y)$

$$\Rightarrow \quad \frac{x-2}{x-3}=\frac{y-2}{y-3}$$

$\Rightarrow xy-3x-2y+6=xy-3y-2x+6$

$\Rightarrow x=y, \forall\, x, y \in A$

So, f is an injective mapping.

Again, let y be an orbitary element in B, then

$$f(x)=y$$

$$\Rightarrow \quad \frac{x-2}{x-3}=y$$

$$\Rightarrow \quad x=\frac{3y-2}{y-1}$$

Clearly, $\forall\, y \in B, x=\frac{3y-2}{y-1} \in A$, thus for all $y \in B$ there exists $x \in A$ such that

$$f(x)=f\left(\frac{3y-1}{y-1}\right)=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}=y$$

Thus, every element in the codomain B has its preimage in A, so f is a surjection. Hence, $f{:}A\rightarrow B$ is bijective.

12 **(a)**

$f(x)$ is defined for

$\sin x \geq 0$ and $1+\sqrt[3]{\sin x} \neq 0$

$\Rightarrow \sin x \geq 0$ and $\sin x \neq -1$

$\Rightarrow \sin x \geq 0$

$\Rightarrow x \in [2n\,\pi,(2n+1)\pi], n \in Z$

$\Rightarrow D=\bigcup\limits_{n\,\in\, Z}[2n\,\pi,(2n+1)\pi]$

Clearly, it contains the interval $(0,\pi)$

13 **(a)**

$fog\,(x)=f(g(x))=f(3x-1)=3(3x-1)^2+2=27x^2-18x+5$

14 **(c)**

We have,

$$|x|=\begin{cases}x, & x\geq 0\\ x, & x<0\end{cases} \Rightarrow |x|-x=\begin{cases}0, & x\geq 0\\ -2\,x, & x<0\end{cases}$$

Hence, domain of $f(x)\frac{1}{\sqrt{|x|-x}}$ is the set of all negative real numbers, i.e., $(-\infty,0)$

16 **(c)**

$$\begin{aligned} gof(x) &= g\{f(x)\} \\ &= g(x^2-1)=(x^2-1+1)^2 \\ &= x^4 \end{aligned}$$

17 **(d)**

$$\sum_{r=1}^{n} f(r)=f(1)+f(2)+f(3)+\ldots+f(n)$$

$= f(1) + 2f(1) + 3f(1) + \dots nf(n)$

[since, $f(x+y) = f(x) + f(y)$]

$= (1 + 2 + 3 + \dots + n)f(1) = f(1)\sum n$

$= \frac{7n(n+1)}{2}$ [$\because f(1) = 7$ (given)]

18 **(c)**

Given, $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $(x-2)(x-1)$

$\therefore \quad f(2) = 2(2)^4 - 13(2)^2 + a(2) + b = 0$

$\Rightarrow \quad 2a + b = 20$ ...(i)

And $\quad f(1) = 2(1)^4 - 13(1)^2 + a + b = 0$

$\Rightarrow \quad a + b = 11$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 9, \quad b = 2$

19 **(d)**

We have, $f(x) = \frac{x^2-8}{x^2+2}$

Clearly, $f(-x) = f(x)$. Therefore, f is not one-one

Again,

$$f(x) = \frac{x^2-8}{x^2+2} = 1 - \frac{10}{x^2+2}$$

$\Rightarrow f(x) < 1 \quad$ for all $x \in R$

$\Rightarrow$ Range $f \neq$ Co-domain of f *i.e.* R.

So, f is not onto. Hence, f is neither one-one nor onto

20 **(b)**

$\sin^{-1}(x-3)$ is defined for the values of x satisfying

$-1 \leq x - 3 \leq 1 \Rightarrow 2 \leq x \leq 4 \Rightarrow x \in [2, 4]$

$\sqrt{9-x^2}$ is defined for the values of x satisfying

$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0 \Rightarrow x \in [-3, 3]$

Also, $\sqrt{9-x^2} = 0 \Rightarrow x = \pm 3$

Hence, the domain of $f(x)$ is $[2, 4] \cap [-3, 3] - \{-3, 3\} = [2, 3)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	B	B	B	B	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	C	D	C	D	C	D	B