

Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

Given, $f(x) = \tan^{-1} \frac{2x}{1-x^2} = 2 \tan^{-1} x (x^2 < 1)$

Since, $x \in (-1, 1)$.

$$\Rightarrow \tan^{-1} x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$\Rightarrow 2 \tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\text{So, } f(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

2 **(a)**

Let $y = f(x) = x^3$

$$\therefore x = y^{1/3}$$

$$\Rightarrow f^{-1}(x) = x^{1/3}$$

$$\therefore f^{-1}(8) = (8)^{1/3} = 2$$

3 **(d)**

For $f(x) = \log_{\frac{x-2}{x+3}} 2$ to exist, we must have

$$\frac{x-2}{x+3} > 0 \text{ and } \frac{x-2}{x+3} \neq 1 \Rightarrow x < -3 \text{ or } x > 2, x \neq -3, x \neq 2$$

For $g(x) = \frac{1}{\sqrt{x^2 - 9}}$ to exist, we must have

$$x^2 - 9 > 0 \Rightarrow x < -3 \text{ or } x > 0$$

Thus, $f(x)$ and $g(x)$ both do not exist for $-3 < x < 2$, i.e., for $x \in (-3, 2)$

4 **(b)**

For choice (a), we have

$$f(x) = f(y), x, y \in [-1, \infty)$$

$$\Rightarrow |x + 1| = |y + 1| \Rightarrow x + 1 = y + 1 \Rightarrow x = y$$

So, f is an injection

For choice (b), we have

$$g(2) = \frac{5}{2} \text{ and } g(1/2) = \frac{5}{2}$$

$$\therefore 2 \neq \frac{1}{2} \text{ but } g(2) = g(1/2)$$

Thus, $g(x)$ is not injective

It can be easily seen that choices $h(x)$ and $k(x)$ are injections

5 **(b)**



We have

$$f(n) = \begin{cases} 2 & \text{if } n = 3k, \quad k \in \mathbb{Z} \\ 10 & \text{if } n = 3k + 1, \quad k \in \mathbb{Z} \\ 0 & \text{if } n = 3k + 2, \quad k \in \mathbb{Z} \end{cases}$$

For $f(n) > 2$, we take $n = 3k + 1, k \in \mathbb{Z}$

$$\Rightarrow n = 1, 4, 7$$

$$\therefore \text{Required set } \{n \in \mathbb{Z}; f(n) > 2\} = \{1, 4, 7\}$$

6 **(b)**

$$\text{Let } y = \frac{2x-1}{x+5}$$

$$\Rightarrow x = \frac{5y+1}{2-y}$$

$$\therefore f^{-1}(x) = \frac{5x+1}{2-x}, x \neq 2$$

7 **(b)**

We have,

$$f(a+x) = b + [b^3 + 1 - 3b^2f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3]^{1/3} \text{ for all } x \in R$$

$$\Rightarrow f(a+x) = b + [1 + \{b - f(x)\}^3]^{1/3} \text{ for all } x \in R$$

$$\Rightarrow f(a+x) - b = [1 - \{f(x) - b\}^3]^{1/3} \text{ for all } x \in R$$

$$\Rightarrow g(a+x) = [1 - \{g(x)\}^3]^{1/3} \text{ for all } x \in R,$$

Where $g(x) = f(x) - 1$

$$\Rightarrow g(2a+x) = [1 - \{g(a+x)\}^3]^{1/3} \text{ for all } x \in R$$

$$\Rightarrow g(2a+x) = [1 - \{1 - (g(x))^3\}]^{1/3} \text{ for all } x \in R$$

$$\Rightarrow g(2a+x) = g(x) \text{ for all } x \in R$$

$$\Rightarrow f(2a+x) - 1 = f(x) - 1 \text{ for all } x \in R$$

$$\Rightarrow f(2a+x) = f(x) \text{ for all } x \in R$$

$\Rightarrow f(x)$ is periodic with period $2a$

8 **(a)**

Given a set containing 10 distinct elements and $f:A \rightarrow A$. Now, every element of a set A can make image in 10 ways.

\therefore Total number of ways in which each element make images $= 10^{10}$.

9 **(c)**

$$\text{Given, } f\left(\frac{p}{q}\right) = \sqrt{p^2 - q^2}, \text{ for } \frac{p}{q} = Q$$

If $p < q$, then $f\left(\frac{p}{q}\right)$ is not real.

Hence, statement I is false while statement II is true.

10 **(c)**

The given function is defined when $x^2 - 1; 3 + x > 0$ and $3 + x \neq 1$

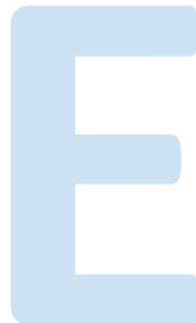
$$\Rightarrow x^2 > 1; \quad 3 + x > 0 \quad \text{and} \quad x \neq -2$$

$$\Rightarrow -1 > x > 1; \quad x > -3, \quad x \neq -2$$

\therefore Domain of the function is

$$D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$$

391 **(a)**



Let x and y be two arbitrary elements in A .

Then, $f(x) = f(y)$

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$

$$\Rightarrow x = y, \forall x, y \in A$$

So, f is an injective mapping.

Again, let y be an arbitrary element in B , then

$$f(x) = y$$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x = \frac{3y-2}{y-1}$$

Clearly, $\forall y \in B, x = \frac{3y-2}{y-1} \in A$, thus for all $y \in B$ there exists $x \in A$ such that

$$f(x) = f\left(\frac{3y-1}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = y$$

Thus, every element in the codomain B has its preimage in A , so f is a surjection. Hence, $f: A \rightarrow B$ is bijective.

12 (a)

$f(x)$ is defined for

$$\sin x \geq 0 \text{ and } 1 + \sqrt[3]{\sin x} \neq 0$$

$$\Rightarrow \sin x \geq 0 \text{ and } \sin x \neq -1$$

$$\Rightarrow \sin x \geq 0$$

$$\Rightarrow x \in [2n\pi, (2n+1)\pi], n \in \mathbb{Z}$$

$$\Rightarrow D = \bigcup_{n \in \mathbb{Z}} [2n\pi, (2n+1)\pi]$$

Clearly, it contains the interval $(0, \pi)$

13 (a)

$$fog(x) = f(g(x)) = f(3x-1) = 3(3x-1)^2 + 2 = 27x^2 - 18x + 5$$

14 (c)

We have,

$$|x| = \begin{cases} x, & x \geq 0 \\ x, & x < 0 \end{cases} \Rightarrow |x| - x = \begin{cases} 0, & x \geq 0 \\ -2x, & x < 0 \end{cases}$$

Hence, domain of $f(x) = \frac{1}{\sqrt{|x| - x}}$ is the set of all negative real numbers, i.e., $(-\infty, 0)$

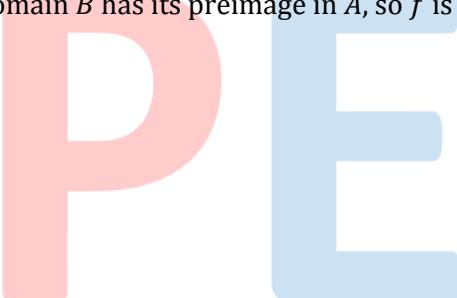
16 (c)

$$gof(x) = g\{f(x)\}$$

$$= g(x^2 - 1) = (x^2 - 1 + 1)^2 \\ = x^4$$

17 (d)

$$\sum_{r=1}^n f(r) = f(1) + f(2) + f(3) + \dots + f(n)$$



$$\begin{aligned}
 &= f(1) + 2f(1) + 3f(1) + \dots + nf(n) \\
 &\quad [\text{since, } f(x+y) = f(x) + f(y)] \\
 &= (1+2+3+\dots+n)f(1) = f(1) \sum n \\
 &= \frac{7n(n+1)}{2} \quad [\because f(1) = 7 \text{ (given)}]
 \end{aligned}$$

18 **(c)**

Given, $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by

$$(x-2)(x-1)$$

$$\therefore f(2) = 2(2)^4 - 13(2)^2 + a(2) + b = 0$$

$$\Rightarrow 2a + b = 20 \quad \dots(\text{i})$$

$$\text{And } f(1) = 2(1)^4 - 13(1)^2 + a + b = 0$$

$$\Rightarrow a + b = 11 \quad \dots(\text{ii})$$

On solving Eqs. (i) and (ii), we get

$$a = 9, \quad b = 2$$

19 **(d)**

We have, $f(x) = \frac{x^2 - 8}{x^2 + 2}$

Clearly, $f(-x) = f(x)$. Therefore, f is not one-one

Again,

$$f(x) = \frac{x^2 - 8}{x^2 + 2} = 1 - \frac{10}{x^2 + 2}$$

$$\Rightarrow f(x) < 1 \quad \text{for all } x \in R$$

$$\Rightarrow \text{Range } f \neq \text{Co-domain of } f \text{ i.e. } R.$$

So, f is not onto. Hence, f is neither one-one nor onto

20 **(b)**

$\sin^{-1}(x-3)$ is defined for the values of x satisfying

$$-1 \leq x-3 \leq 1 \Rightarrow 2 \leq x \leq 4 \Rightarrow x \in [2, 4]$$

$\sqrt{9-x^2}$ is defined for the values of x satisfying

$$9-x^2 \geq 0 \Rightarrow x^2-9 \leq 0 \Rightarrow x \in [-3, 3]$$

$$\text{Also, } \sqrt{9-x^2} = 0 \Rightarrow x = \pm 3$$

Hence, the domain of $f(x)$ is $[2, 4] \cap [-3, 3] - \{-3, 3\} = [2, 3]$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	B	B	B	B	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	C	D	C	D	C	D	B

P E