DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

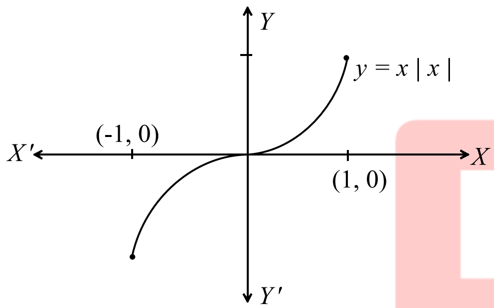
Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

We have,

$$f(x)=x|x|=\begin{cases} x^2, & 0\le x\le 1 \\ -x^2, & -1\le x<0 \end{cases}$$

The graph of $f(x)$ is as shown below. Clearly, it is a bijection



2 **(b)**

Foe domain of given function

$$-1\le \log_2 \frac{x^2}{2}\le 1$$

$\Rightarrow\ 2^{-1}\le \frac{x^2}{2}\le 2 \Rightarrow 1\le x^2\le 4$

$\Rightarrow\ |x|\le 2$ and $|x|\ge 1$

$\Rightarrow\ x\in[-2,2]-(-1,1)$

3 **(c)**

Given, $f(x)=ax+b,\quad g(x)=cx+d$

$\because\ f\{g(x)\}=g\{f(x)\}$

$\Rightarrow\ f(cx+d)=g(ax+b)$

$\Rightarrow\ a(cx+d)+b=c(ax+b)+d$

$\Rightarrow\ ad+b=bc+d$

$\Rightarrow\ f(d)=g(b)$

4 **(c)**

Since $\phi(x)=\sin^4 x+\cos^4 x$ is periodic with period $\pi/2$

$\therefore f(x)=\sin^4 3x+\cos^4 3x$ is periodic with period $\frac{1}{3}\left(\frac{\pi}{2}\right)=\frac{\pi}{6}$

5 **(b)**

We have,

$f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$

$\therefore fog\ (x) = f(g(x))$

$$\Rightarrow fog\ (x) = f\left(\frac{3x+x^3}{1+3x^2}\right) = \log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right) = \log\left\{\frac{(1+x)^3}{(1-x)^3}\right\}$$

$$\Rightarrow fog\ (x) = \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$$

6 **(b)**

For choice (a), we have

$f(x) = f(y); x, y \in [-1, \infty)$

$\Rightarrow |x+1| = |y+1| \Rightarrow x+1 = y+1 \Rightarrow x = y$

So, f is an injection

For choice (b), we obtain

$g(2) = \frac{5}{2}$ and $g\left(\frac{1}{2}\right) = \frac{5}{2}$

So, $g(x)$ is not injective

It can be easily seen that the functions in choices in options (c) and (d) are injective maps

7 **(b)**

Given, $f(x) = x - [x]$, $g(x) = [x]$ for $x \in R$.

$\therefore \quad f(g(x)) = f([x])$

$= [x] - [x]$

$= 0$

8 **(a)**

We have,

$$f(x) = \sqrt{\frac{\log_{0.3}|x-2|}{|x|}}$$

We observe that $f(x)$ assumes real values, if

$\frac{\log_{0.3}|x-2|}{|x|} \geq 0$ and $|x-2| > 0$

$\Rightarrow \log_{0.3}|x-2| \geq 0$ and $x \neq 0, 2$

$\Rightarrow |x-2| \leq 1$ and $x \neq 0, 2$

$\Rightarrow x \in [1, 3]$ and $x \neq 2 \Rightarrow x \in [1, 2) \cup (2, 3]$

9 **(d)**

Since $g(x) = 3\sin x$ is a many-one function. Therefore, $f(x) - 3\sin x$ is many-one

Also, $-1 \leq \sin x \leq 1$

$\Rightarrow -3 \leq -3\sin x + 3$

$\Rightarrow 2 \leq 5 - 3\sin x \leq 8$

$\Rightarrow 2 \leq f(x) \leq 8 \Rightarrow$ Range of $f(x) = [2, 8] \neq R$

So, $f(x)$ is not onto

Hence, $f(x)$ is neither one-one nor onto

10 **(a)**

We have,

$f(x+2y, x-2y) = xy$ ....(i)

Let $x+2y = u$ and $x-2y = v$. Then,

$x = \frac{u+v}{2}$ and $y = \frac{u-v}{4}$

Substituting the values of x and y in (i), we obtain

$f(u,v) = \frac{u^2-v^2}{2}$ and $f(x,y) = \frac{x^2-y^2}{8}$

11 **(c)**

Given, $f(x) = y = (1-x)^{1/3}$

$\Rightarrow \quad y^3 = 1-x$

$\Rightarrow \quad x = 1-y^3$

$\therefore \quad f^{-1}(x) = 1-x^3$

12 **(a)**

We have, $f(x+2y, x-2y) = xy$ ...(i)

Let $x+2y = u$ and $x-2y = v$

Then, $x = \frac{u+v}{2}$ and $y = \frac{u-v}{4}$

Subtracting the values of x and y in Eq. (i), we obtain

$f(u, v) = \frac{u^2-v^2}{8} \Rightarrow f(x, y) = \frac{x^2-y^2}{8}$

13 **(d)**

Given, $f(x) = 5^{x(x-4)}$ for $f{:}[4, \infty[\rightarrow [4, \infty[$

At $x = 4$

$f(x) = 5^{4(4-4)} = 1$

Which is not lie in the interval $[4, \infty[$

$\therefore$ Function is not bijective.

Hence, $f^{-1}(x)$ is not defined.

14 **(b)**

Given, $f(x) = x^3 + 3x - 2$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 + 3$

Put $f'(x) = 0 \Rightarrow 3x^2 + 3 = 0$

$\Rightarrow \quad x^2 = -1$

$\therefore \quad f(x)$ is either increasing or decreasing.

At $x = 2$, $f(2) = 2^3 + 3(2) - 2 = 12$

At $x = 3$, $f(3) = 3^3 + 3(3) - 2 = 34$

$\therefore f(x) \in [12, 34]$

15 **(b)**

We have,

$f(\theta) = \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\therefore f(\theta)$ is periodic with period $\frac{2\pi}{2} = \pi$

16 **(c)**

Since, period of $\cos nx = \frac{2\pi}{n}$

And period of $\sin\left(\frac{x}{n}\right) = 2n\pi$

$\therefore$ Period of $\frac{\cos nx}{\sin\left(\frac{x}{n}\right)}$ is $2n\pi$

$\Rightarrow\ 2n\pi = 4\pi\ \Rightarrow\ n = 2$

17 **(c)**

Given, $f(x) = x^3 + 5x + 1$

Now, $f'(x) = 3x^2 + 5 > 0, \forall x \in R$

$\therefore f(x)$ is strictly increasing function.

$\therefore f(x)$ is one-one function.

Clearly, $f(x)$ is a continous function and also increasing on R,

$\lim_{x\to-\infty} f(x) = -\infty\ and\ \lim_{x\to\infty} = \infty$

$\therefore f(x)$ takes every value between $-\infty$ and ∞

Thus, $f(x)$is onto function.

18 **(c)**

The function $f(x) = \frac{1}{2 - \cos 3x}$ is defined for all $x \in R$. Therefore, domain of $f(x)$ is R

Let $f(x) = y$. Then,

$\frac{1}{2 - \cos 3x} = y$ and $y > 0$

$\Rightarrow 2 - \cos 3x = \frac{1}{y}$

$\Rightarrow \cos 3x = \frac{2y - 1}{y} \Rightarrow x = \frac{1}{3}\cos^{-1}\left(\frac{2y-1}{y}\right)$

Now,

$x \in R$, if

$-1 \le \frac{2y-1}{y} \le 1$

$\Rightarrow -1 \le 2 - \frac{1}{y} \le 1$

$\Rightarrow -3 \le -\frac{1}{y} \le -1$

$\Rightarrow 3 \ge \frac{1}{y} \ge 1 \Rightarrow \frac{1}{3} \le y \le 1 \Rightarrow y \in [1/3, 1]$

19 **(c)**

Given, $A = \{2, 3, 4, 5, \ldots, 16, 17, 18\}$

And $(a, b) = (c, d)$

$\therefore$ Equivalence class of (3, 2) is

$\{(a, b) \in A \times A : (a, b) R\ (3, 2)\}$

$= \{(a, b) \in A \times A : 2a = 3b\}$

$= \left\{(a, b) \in A \times A : b = \dfrac{2}{3} a\right\}$

$\left\{\left(a, \dfrac{2}{3} a\right) : a \in A \times A\right\}$

$= \{(3, 2), (6, 4), (9, 6), (12, 8), (15, 10), (18, 12)\}$

$\therefore$ Number of ordered pairs of the equivalence class=6.

20 **(c)**

Given function is $f(n) = 8 - {}^{n}P_{n-4}$, $4 \le n \le 6$. It is defined, if

1. $8 - n > 0 \Rightarrow n < 8$ ...(i)
2. $n - 4 \ge 0 \Rightarrow n \ge 4$ ...(ii)
3. $n - 4 \le 8 - n \Rightarrow n \le 6$ ...(iii)

From Eqs. (i), (ii) and (iii), we get $n = 4, 5, 6$

Hence, range of $f(n) = \{{}^{4}P_0, {}^{3}P_1, {}^{2}P_2\} = \{1, 3, 2\}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	C	C	B	B	B	A	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	D	B	B	C	C	C	C	C