DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :4

Topic :-PROBABILITY

1 **(d)**

Since there are r cars in N places, total number of selection of places out of $N-1$ places for $r-1$ cars (excepting the owner's car) is,

$$^{N-1}C_{r-1}=\frac{(N-1)!}{(r-1)!(N-r)!}$$

If neighboring places are empty, then $r-1$ cars must be parked in $N-3$ places. So, the favourable number of cases is

$$^{N-3}C_{r-1}=\frac{(n-3)!}{(r-1)!(N-r-2)!}$$

Therefore, the required probability is

$$\frac{(N-3)!}{(r-1)!(N-r-2)!}\times\frac{(r-1)!(N-r)!}{(N-1)!}$$

$$=\frac{(N-1)(N-r-1)}{(N-1)(N-2)}=\frac{^{N-r}C_2}{^{N-1}C_2}$$

2 **(a)**

We have,

$n(S)=\ ^{64}C_3$

Let 'E' be the event selecting 3 squares which form the letter 'L'.

The number of ways of selecting squares consisting of 4 unit squares is $7\times7=49$

Each square with 4 unit squares form 4 L-shapes consisting of 3 unit squares

$\therefore n(E)=7\times7\times4=196$

$$\therefore\ \ P(E)=\frac{196}{^{64}C_3}$$

3 **(a)**

Required probability $=\frac{\text{No.of favourite cases}}{\text{Total no.of exhaustive cases}}$

$$=\frac{3}{3\times3\times3}-\frac{1}{9}$$

4 **(b)**

The sum of the digits can be 7 in the following ways: 07, 16, 25, 34, 43, 52, 61, 70

$\therefore (A=7)=\{07, 16, 25, 34, 43, 52, 61, 70\}$

Similarly,

$(B=0)=\{00, 01, 02, \ldots 10, 20, 30, \ldots, 90\}$
Thus,
$(A=7)\cap(B=0)=\{09, 70\}$
$\therefore\ P((A=7)\cap(B=0))=\frac{2}{100}, P((B=0))=\frac{19}{100}$
Hence,
$P(A=7 \mid B=0)=\frac{P((A=7)\cap(B=0))}{P(B=0)}$
$=\frac{\frac{2}{100}}{\frac{19}{100}}=\frac{2}{19}$
5 **(d)**
A: Doctor finds a rash
B_1: Child has measles
S: Sick children
$P(S/F)=0.9$
B_2: Child has flu $\Rightarrow P(B_2)=9/10$
$P(S/M)=0.10$
$P(A/B_1)=0.95$
$P(R/M)=0.95$
$P(A/B_2)=0.08$
$P(R/F)=0.08$
$P(B_1/A)=\frac{0.1\times 0.95}{0.1\times 0.95+0.9\times 0.08}$
$=\frac{0.095}{0.095+0.072}$
$=\frac{0.095}{0.167}=\frac{95}{167}$
6 **(c)**
Let A denote the event that target is hit when x shells are filled at point I. Let P_1 and P_2 denote the event that the target is at point I and II, respectively. We have $P(P_1)=8/9, P(P_2)=1/9, P(A/P_1) = 1-(1/2)^x, P(A/P_2)=1-(1/2)^{55-x}$
Now from total probability theorem
$P(A)=P(P_1)P(A/P)+P(P_2)P(A/P_2)$
$=\frac{1}{9}\left(8-8\left(\frac{1}{2}\right)^x+1-\left(\frac{1}{2}\right)^{55-x}\right)$
$=\frac{1}{9}\left(9-8\left(\frac{1}{2}\right)^x-\left(\frac{1}{2}\right)^{55-x}\right)$
Now,
$\frac{dP(A)}{dx}=\frac{1}{9}\left(-8\left(\frac{1}{2}\right)^x \text{In}\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^{55-x}\text{In}\left(\frac{1}{2}\right)\right)$
(Note that in this step, it is being assumed that $x\in R^+$)

$$=\frac{1}{9}In\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{55-x}\left(1-\left(\frac{1}{2}\right)^{2x-58}\right)$$

>0 if $x<29$

<0 if $x>29$

Therefore, $P(A)$ is maximum at $x=29$. Thus, '29' shells must be fired at point I

7 **(d)**

In the first case, the urn constains 3 red and n white balls. The probability that colour of both the balls matches is

$$\frac{{}^3C_2\,{}^nC_2}{{}^{n+3}C_2}=\frac{1}{2}$$

$$\Rightarrow\frac{6+n(n-1)}{(n+3)(n+2)}=\frac{1}{2}$$

$$\Rightarrow 2(n^2-n+6)=n^2+5n+6$$

$$\Rightarrow vn^2-7n+6=0$$

$\Rightarrow n=1$ or 6 (1)

In the second case,

$$\frac{3}{n+3}\frac{3}{n+3}+\frac{n}{n+3}\frac{n}{n+3}=\frac{5}{8}$$

Solving, we get

$$n^2-10n+9=0$$

$\Rightarrow n=9$ or 1 (2)

From Eqs.(1) and (2), we have $n=1$

8 **(b)**

$$P(E)+P(E')=1=1+\lambda+\lambda^2+(1+\lambda)^2$$

$$\Rightarrow 2\lambda^2+3\lambda+1=0$$

$$\Rightarrow(2\lambda+1)+(\lambda+1)=0$$

$$\Rightarrow\lambda=-1,\ -\frac{1}{2}$$

Then, $P(E)=1+(-1)+(-1)^2=1$ (not possible)

$$\Rightarrow P(E)=1+\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)^2=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

9 **(a)**

Consider the following events:

A: ball drawn is black

E_1:bag I is chosen

E_2: bag II is chosen

E_3: bag III is chosen

Then,

$$P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$$

$$P(A/E_1)=\frac{3}{5},P(A/E_2)=\frac{1}{5},\qquad P(A/E_3)=\frac{7}{10}$$

Therefore, the required probability is

$P(E_3/A)$

$$= \frac{P(E_3)P(A/E_3)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P(A/E_3)} = \frac{7}{15}$$

10 **(a)**

For ranked 1 and 2 players to be winners and runners up, respectively, they should not be paired with each other in any round. Therefore, the required probability is

$$30/31 \times 14/15 \times 6/7 \times 2/3 = 16/31$$

11 **(c)**

Total number of the students is 80. Total number of girls is 25. Total number of boys is 55. There are 10 rich, 70 poor, 20 intelligent students in the class. Therefore, required probability is

$$\frac{1}{4} \times \frac{1}{8} \times \frac{25}{80} = \frac{5}{512}$$

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12 **(d)**

The probability that the first critic favours the book is

$$P(E_1) = \frac{5}{5+2} - \frac{5}{7}$$

The probability that the second critic favours the book is

$$P(E_2) = \frac{4}{4+3} = \frac{4}{7}$$

The probability that the third critic favours the book is

$$P(E_3) = \frac{3}{3+4} = \frac{3}{7}$$

Majority will be in favour of the book if at least two crities favour the book. Hence, the probability is

$$P(E_1 \cap E_2 \cap \overline{E_3}) + P(E_1 \cap \overline{E_2} \cap E_3) + P(\overline{E_1} \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

$$= P(E_1)P(E_2)P(\overline{E_3}) + P(E_1)P(\overline{E_2})P(E_3) + P(\overline{E_1} \cap E_2 \cap E_3) + P(E_1)P(E_2)P(E_3)$$

$$= \frac{5}{7} \times \frac{4}{7} \times \left(1 - \frac{3}{7}\right) + \frac{5}{7} \times \left(1 - \frac{4}{7}\right) \times \frac{3}{7} + \left(1 - \frac{5}{7}\right) \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}$$

$$= \frac{209}{343}$$

13 **(b)**

The required probability is

$$\frac{n^2}{{}^{2n}C_2} \frac{(n-1)^2}{{}^{2n-2}C_2} \frac{(n-2)^2}{{}^{2n-4}C_2} \dots \frac{2^2}{{}^4C_2} \frac{1^2}{{}^2C_2}$$

$$= \frac{(1 \times 2 \times 3 \times 4 \times \dots \times (n-1)n^2)}{\frac{(2n)!}{2^n}} = \frac{2^n (n)^2}{(2n)!} = \frac{2^n}{{}^{2n}C_n}$$

14 **(c)**

$f'(x) = 3x^2 + 2ax + 9$

$y = f(x)$ is increasing

$\Rightarrow f'(x) \geq 0, \forall\ x$ and for $f'(x) = 0$ should not form an interval

$\Rightarrow (2a)^2 - 4 \times 3 \times b \leq 0 \Rightarrow a^2 - 3b \leq 0$

This is true for exactly 16 ordered pairs $(a, b) \leq a, b \leq 6$, namely (1,1), (1, 2), (1, 3)(1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 6 and (4, 6). Thus, the required probability is 16/36=4/9

15 **(b)**

There are 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The last two digits can be dialed in ${}^{10}P_2 = 90$ ways out of which only one way is favourable, thus, the required probability is 1/90

16 **(a)**

Let E_i denote the event that the bag contains i black and $(10 - i)$ white balls $(i = 0, 1, 2, ...,10)$. Let A denote the event that the three balls drawn at random from the bag are black. We have,

$P(E_i) = \frac{1}{11}(i = 0,1,2,...,10)$

$P(A/E_i) = 0$ for $i = 0, 1, 2$ and $P(A/E_i) = iC_3/\ {}^{10}C_3$ for $i \geq 3$

$\Rightarrow P(A) = \frac{1}{11} \times \frac{1}{{}^{10}C_3}[\ {}^3C_3 + {}^4C_3 + ... + {}^{10}C_3]$

But ${}^3C_3 + {}^4C_3 + {}^5C_3 + ... + {}^{10}C_3 = {}^4C_4 + {}^4C_3 + {}^5C_3 + ... + {}^{10}C_3$

$= {}^5C_4 + {}^5C_3 + {}^6C_3 + ... + {}^{10}C_3$

$\vdots$

$= {}^{11}C_4$

$\Rightarrow p(A) = \frac{1}{11} \times \frac{1}{{}^{10}C_3} \times {}^{11}C_4$

$= \frac{\frac{11 \times 10 \times 9 \times 8}{4!}}{11 \times \frac{10 \times 9 \times 8}{3!}} = \frac{1}{4}$

$\therefore P\left(\frac{E_9}{A}\right) = \frac{P(E_9)P(A/E_9)}{P(A)}$

$= \frac{\frac{1}{11} \times \frac{{}^9C_3}{{}^{10}C_3}}{\frac{1}{4}}$

$= \frac{14}{55}$

17 **(b)**

Let $P(m), P(p), P(c)$ be the probability of selecting a book of maths, physics and chemistry, respectively, clearly,

$P(m) = P(p) = P(c) = \frac{1}{3}$

Again let $P(s_1)$ and $P(s_2)$ be the probability that he solves the first as well as second problem, respectively. Then,

$P(s_1) = P(m) \times p\left(\frac{s_1}{m}\right) + p(p) \times p\left(\frac{s_1}{p}\right) + P(c) \times P\left(\frac{s_1}{c}\right)$

$\Rightarrow P(s_1) = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{4}{5} = \frac{19}{30}$

Similarly,

$$P(S_2)=\frac{1}{3}\left(\frac{1}{2}\right)^2+\frac{1}{3}\times\left(\frac{3}{5}\right)^2+\frac{1}{3}\times\left(\frac{4}{5}\right)^2=\frac{125}{300}$$

$$\Rightarrow P\left(\frac{S_2}{S_1}\right)=\frac{\frac{125}{300}}{\frac{19}{30}}=\frac{25}{38}$$

18 **(b)**

Consider the following events:

A: getting a card with mark I in first draw

B:getting card with mark I in second draw

C: getting a card with mark T in this draw

Then, the required probability is

$P(A\cap B\cap C)=P(A)P(B/A)P(C/A\cap B)$

$$=\frac{10}{20}\times\frac{9}{19}\times\frac{10}{18}=\frac{5}{38}$$

19 **(d)**

We have,

$$x+\frac{100}{x}>50$$

$\Rightarrow x^2+100>50\,x$

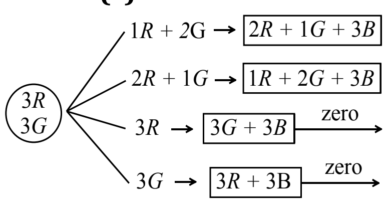
$\Rightarrow (x-25)^2>525$

$\Rightarrow x-25<\sqrt{525}\ \ or\ \ x-25>\sqrt{525}$

$\Rightarrow x<25-\sqrt{525}\ or\ 25+\sqrt{525}$

As x is a positive integer and $\sqrt{525}=22.91$, we must have $x\le 2$ or $x\ge 48$. Thus, the favourable number of cases is $2+53=55$. Hence, the required probability is 55/100=11/20

20 **(c)**



The required probability is

$$\frac{^3C_1\ ^3C_2}{^6C_3}\frac{^2C_1\ ^1C_1\ ^3C_1}{^6C_3}+\frac{^3C_2\ ^3C_1}{^6C_3}\frac{^1C_1\ ^2C_1\ ^3C_1}{^6C_3}$$

$$=2\frac{9}{20}\times\frac{6}{20}$$

$$=\frac{27}{100}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	A	B	D	C	D	B	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	B	C	B	A	B	B	D	C

PE