DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-PROBABILITY

1 **(b)**

Total number of ways of distribution is 4^5

$\therefore n(S) = 4^5$

Total number of ways of distribution so that each child gets at least one game is

$4^5 - {}^4C_1\, 3^5 + {}^4C_2\, 2^5 - {}^4C_3 = 1024 - 4 \times 243 + 6 \times 32 - 4 = 240$

$\therefore\ n(E) = 240$

Therefore, the required probability is

$$\frac{n(E)}{n(S)} = \frac{240}{4^5} = \frac{15}{64}$$

2 **(c)**

We know that the number of subsets of a set containing n elements is 2^n. Therefore, the number of ways of choosing P and Q is

${}^{2^n}C_1 \times {}^{2^n}C_1 = 2^n \times 2^n = 4^n$

Out of n elements, m elements are chosen and then from the remaining $n-m$ elements either an element belongs to P or$Q$. But not both P and$Q$. Suppose P contains r elements from the remaining $n-m$ elements. Then, Q may contain any number of elements from the remaining $(n-m)-r$ elemts. Therefore, P and Q can be chosen in ${}^{n-m}C_r 2^{(n-m)-r}$ ways

But r can vary from 0 to $n-m$. So, in general the number of ways in which P and Q can be chosen is

$$\left(\sum_{r=0}^{n-m} {}^{n-m}C_r 2^{(n-m)-r}\right) {}^nC_m = (1+2)^{n-m}\, {}^nC_m = {}^nC_m 3^{n-m}$$

Hence, the required probability is ${}^nC_m\, 3^{n-m}/4^n$

3 **(a)**

The total number of ways of making the second draw is ${}^{10}C_5$

The number of draw of 5 balls containing 2 balls common with first draw of 6 balls is ${}^6C_2\, {}^4C_3$.

Therefore, the probability is

$$\frac{{}^6C_2\, {}^4C_3}{{}^{10}C_5} = \frac{5}{21}$$

4 **(c)**

The total number of digits in any number at the unit's place is 10

$\therefore\ n(S) = 10$

If the last digit in product is 1,3, 5 or 7, then it is necessary that the last digit in each number must be 1,3,5 or 7

$\therefore n(A) = 4$

$\therefore P(A) = \frac{4}{10} = \frac{2}{5}$

Hence, the required probability is $(2/5)^4 = 16/625$

5 **(d)**

$P(A) = \frac{2}{5}$

For independent events,

$P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B) \leq \frac{2}{5}$

$\Rightarrow P(A \cap B) = \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$

[Maximum 4 outcomes may be in $P(A \cap B)$]

1. When $P(A \cap B) = \frac{1}{10}$

$\Rightarrow P(A).P(B) = \frac{1}{10}$

$\Rightarrow P(B) = \frac{1}{10} \times \frac{5}{2} = \frac{1}{4}$, not possible

2. When $P(A \cap B) = \frac{2}{10} \Rightarrow \frac{2}{5} \times P(B) = \frac{2}{10}$

$\Rightarrow P(B) = \frac{5}{10}$, outcomes of $B = 5$

3. When $P(A \cap B) = \frac{3}{10}$

$\Rightarrow P(A)P(B) = \frac{3}{10}$

$\Rightarrow \frac{2}{5} \times P(B) = \frac{3}{10}$

$P(B) = \frac{3}{4}$, not possible

4. When $P(A \cap B) = \frac{4}{10}$

$\Rightarrow P(A).P(A) = \frac{4}{10}$

$\Rightarrow P(B) = 1$, outcomes of $B = 10$

6 **(d)**

A person can have his/her birthday on any one of the seven days of the week. So 5 persons can have their birthdays in 7^5 ways. Out of 5, three persons can have their birthday on days other than Sunday in 6^3 ways and other 2 on Sundays. Hence, the required probability is

$$\frac{{}^5C_2 \times 6^3}{7^5} = \frac{10 \times 6^3}{7^5}$$

(Note that 2 persons can be selected out of 5 in ${}^5C_2$ ways)

7 **(a)**

$$P(B_1) = \frac{{}^6C_1}{{}^{10}C_1} = \frac{6}{10} = \frac{3}{5}$$

$$P(B_2/B_1) = \frac{5}{9}(B_2 = \text{black}) \therefore P(B_1 \cap B_2) = P(B_1)P(B_2/B_1) = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$$

78 **(d)**

Three-digit numbers are 100,101, …999. Total number of such numbers is 900. The three-digit numbers (which have all same digits) are 111,222, 333, …, 999. Favourable number of cases is 9. Therefore, the required probability is 9/900=1/100

79 **(b)**

Let E be the event of getting 1 on a die

$\Rightarrow P(E) = \frac{1}{6}$ and $P(\bar{E}) = \frac{5}{6}$

$\therefore P(\text{first time 1 occurs at the even throw})$

$= t_2$ or t_4 or t_6 or t_8… and so on.

$= \{P(\bar{E}_1).P(E_2)\} + \{P(\bar{E}_1)P(\bar{E}_2)P(\overline{E_3})P(E_4)\} + \dots\infty$

$= \left(\frac{5}{6}.\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \dots\infty$

$$= \frac{\frac{5}{36}}{1 - \frac{25}{36}} = \frac{5}{11}$$

10 **(b)**

$P(S \cap F) = 0.006$, where S is the event that the motor cycle is stolen and F is the event that the motor cycle is found. Therefore,

$P(S) = 0.0015$

$$P(F/S) = \frac{P(F \cap S)}{P(S)} = \frac{6 \times 10^{-4}}{15 \times 10^{-4}} = \frac{2}{5}$$

11 **(d)**

The total number of ways of choosing 11 players out of 15 is ${}^{15}C_{11}$. A team of 11 players containing at least 3 bowlers can be chosen in the following mutually exclusive ways:

1. Three bowlers out of 5 bo0wlers and 8 other players out of the remaining 10 players

2. Four bowlers out of 5 bowlers and 7 other players out of the remaining 10 players

3. Five bowlers out of 5 bowlers and 6 other players out of the remaining 10 players

So, required probability is

$$P(I)+P(II)+P(III)=\frac{{}^{5}C_3\times{}^{10}C_8}{{}^{15}C_{11}}+\frac{{}^{5}C_4\times{}^{10}C_7}{{}^{15}C_{11}}+\frac{{}^{5}C_5\times{}^{10}C_6}{{}^{15}C_{11}}$$

$$=\frac{1260}{1365}=\frac{12}{13}$$

12 **(a)**

Let the number selected by xy. Then

$x+y=9, 0<x, y\leq 9$

And $xy=0 \Rightarrow x=0, y=9$

Or $y=0, x=9$

$$P(x_1=9/x_2=0)=\frac{P(x_1=9\cap x_2=0)}{P(x_2=0)}$$

Now, $P(x_2=0)=\frac{19}{100}$

And $P(x_1=9\cap x_2=0)=\frac{2}{100}$

$$\Rightarrow P(x_1=9/x_2=0)=\frac{2/100}{19/100}=\frac{2}{19}$$

13 **(d)**

Since a, b, c are in A.P>, therefore, $2b=a+c$. The possible cases are tabulated as fallows

b	a	c	Number of ways
1	1	1	1
2	2	2	1
2	1	3	6
3	3	3	1
3	1	5	6
3	2	4	6

Total number of ways is 21. So, required probability is 21/216=7/72

14 **(b)**

We define the following events:

A_1: Selecting a pair of consecutive letters from the word LONDON

A_2: Selecting a pair of consecutive letters from the word CLIFTON

E: Selecting a pair of letters 'ON'

Then, $P(A_1E)=2/5$ as there are 5 pairs of consecutive letters out of which 2 are ON and $P(A_2E)$ $=1/6$ as there are 6 pairs of consecutive letters of which 1 is ON. Therefore, the required probability is

$$P\left(\frac{A_1}{E}\right)=\frac{P(A_1\cap E)}{P(A_1\cap E)+P(A_2\cap E)}=\frac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{6}}=\frac{12}{17}$$

15 **(c)**

The sum is 12 in first three throws if they are (1,5, 6) in any order or (2, 4, 6) in any order or (3,4, 5) in any order. Therefore, the required probability is

$$\frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times 3!+\frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times 3!+\frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times 3!=\frac{3}{20}$$

(because after throwing 1, in the next throw 1 cannot come, etc.)

16 **(d)**

The total number of ways in which papers of 4 students can be checked by seven teachers is 7^4. The number of ways of choosing two teachers out of 7 is ${}^7C_2$. The number of ways in which they can check four papers is 2^4. But this includes two ways in which all the papers will be checked by a single teacher. Therefore, the number of ways in which 4 papers can be checked by exactly two teachers is $2^4 - 2 = 14$. Therefore, the number of favourable ways is $({}^7C_2)(14) = (21)(14)$. Thus, the required probability is $(21)(14)/7^4 = 6/49$

17 **(c)**

A: car met with an accident

B_1: driver was alcoholic, $P(B_1) = 1/5$

B_2: driver was sober,$P(B_2) = 4/5$

$P(A/B_1) = 0.001$; $P(A/B_2) = 0.0001$

$$P\left(\frac{B_1}{A}\right)=\frac{(0.2)(0.001)}{(0.2)(0.001)+(0.8)(0.0001)}=5/7$$

18 **(c)**

Out of 5 horses, only one is the wining horse. The probability that Mr. A selected that losing horse is $4/5 \times 3/4$. Therefore, the required probability is

$$1-\frac{4}{5}\times\frac{3}{4}=1-\frac{3}{5}=\frac{2}{5}$$

19 **(c)**

Suppose, there exist three rational points or more on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Therefore, if (x_1,y_1), (x_2,x_2) and (x_3, y_3) be those three points, then

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$ (1)

$x_2^2 + y_2^2 + 2gx_2 + 2fy_1 + c = 0$ (2)

$x_3^2 + y_3^2 + 2gx_3 + 2fy_3 + c = 0$ (3)

Solving Eqs. (1), (2) and (3), we will get g, f, c as rational. Thus, centre of the circle $(-g. -f)$ is a rational point. Therefore, both the coordinates of the centre are rational numbers. Obviously, the possible values of p are 1, 2. Similarly, the possible values of q are 1, 2. Thus fro this case (p, q) may be chosen in 2×2, i.e., 4 ways. Now, (p, q) can be, without restriction, chosen in 6×6, i.e., 36 ways Hence, the probability that at the most two rational points exist on the circle is

$$(36-4)/36 = 32/36 = 8/9$$

20 **(a)**

The required probability is

$$P(A)=\frac{1}{3}\frac{6}{a^2-4a+10}$$

$$(P(A))_{\max}=\frac{1}{3}\times\frac{6}{6}=\frac{1}{3}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	A	C	D	D	C	D	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	D	B	C	D	C	C	C	A