DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-PROBABILITY

1 **(b)**

The total number of ways in which n persons can sit at a round table is $(n-1)!$. So, total number of cases is $(n-1)!$

Let A and B be two specified persons. Considering these two as one person, the total number of ways in which $n-1$ persons, $n-2$ other persons and one AB can sit at a round table is $(n-2)!$. So, favourable number of cases is $2!(n-2)!$ Thus, the required probability is

$$p=\frac{2!(n-2)!}{(n-1)!}=\frac{2}{n-1}$$

Hence, the required odds are $(1-p):p$ or $(n-3):2$

2 **(b)**

Here $p=19/20$, $q=1/20$, $n=5$, $r=5$. The required probability is

$$^5C_5\left(\frac{19}{20}\right)^5\left(\frac{1}{20}\right)^6=\left(\frac{19}{20}\right)^5$$

3 **(a)**

Let the number of red and blue balls be r and b, respectively. Then, the probability of drawing two red balls is

$$p_1=\frac{^rC_2}{^{r+b}C_2}=\frac{r(r-1)}{(r+b)(r+b-1)}$$

The probability of drawing two blue balls is

$$p_2=\frac{^bC_2}{^{r+b}C_2}=\frac{b(b-1)}{(r+b)(r+b-1)}$$

The probability of drawing one red and one blue ball is

$$p_3=\frac{^rC_1\,^bC_1}{^{r+b}C_2}=\frac{2br}{(r+b)(r+b-1)}$$

By hypothesis, $p_1=5p_2$ and $p_3=6p_2$

$\therefore r(r-1)=5b(b-1)$ and $2br=6b(b-1)$

$\Rightarrow r=6, b=3$

4 **(d)**

Consider two events as follows:

A_1: getting number i on first die

B_1: getting a number more than i on second die
The required probability is

$$P(A_1 \cap B_1) + P(A_2 \cap B_2) + P(A_3 \cap B_3) + P(A_4 \cap B_4) + P(A_5 \cap B_5) = \sum_{i=1}^{5} P(A_i \cap B_i)$$

$$= \sum_{i=1}^{5} P(A_i) P(B_i)$$

[$\because A_i, B_i$ are independent]

$$= \frac{1}{6}\,[P(B_1) + P(B_2) + \ldots + P(B_5)]$$

$$= \frac{1}{6}\left(\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6}\right) = \frac{5}{12}$$

5 **(b)**

Let,

$P(S) = P(1 \text{ or } 2) = 1/3$

$P(F) = P(3 \text{ or } 4 \text{ or } 5 \text{ or } 6) = 2/3$

$P(A \text{ wins}) = P[(S\,S \text{ or } S\,F\,S\,S \text{ or } S\,F\,S\,S\,F\,S\,S \text{ or} \ldots)$

Or $(F\,S\,S \text{ or } F\,S\,F\,S\,S \text{ or} \ldots)]$

$$= \frac{\frac{1}{9}}{1 - \frac{2}{9}} + \frac{\frac{2}{27}}{1 - \frac{2}{9}}$$

$$= \frac{1}{9} \times \frac{9}{7} + \frac{2}{27} \times \frac{9}{7}$$

$$= \frac{1}{7} + \frac{2}{21} = \frac{3+2}{21} = \frac{5}{21}$$

$P(A \text{ winning}) = \frac{5}{21}, P(B \text{ winning}) = \frac{16}{21}$

6 **(a)**

Let $E_1 = 1, 4, 7, \ldots$ (n each)

$E_2 = 2, 5, 8, \ldots$ (n each)

$E_3 = 3, 6, 9, \ldots$ (n each)

x and y belong to (E_1, E_2), (E_2, E_1) or (E_3, E_3). So, the required probability is

$$\frac{n^2 + {}^nC_2}{{}^{3n}C_2} = \frac{1}{3}$$

7 **(c)**

Let A be the event that 11 is picked and B be the event that sum is even. The number of ways of selecting 11 along with one more-odd number is $n(A \cap B) = {}^7C_1$

The number of ways of selecting either two even numbers or selecting two odd numbers is $n(B)$ $= 1 + {}^8C_2$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{7}{29} = 0.24$$

8 **(c)**

18 draws are required for 2 aces means in the first 17 draws, there is one ace and 16 other cards and 18th draw produces an ace. So, the required probability is

$$\frac{{}^{48}C_{16} \times {}^{4}C!}{{}^{52}C_{17}} \times \frac{3}{35} = \frac{561}{15925}$$

9 **(c)**

Given,

$$7a - 9b = 0 \Rightarrow b = \frac{7}{9}a$$

Hence, number of pairs (a, b) can be (9, 7); (18, 14); (27, 21); (36, 28). Hence, the required probability is $4/{}^{39}C_2 = 4/741$

10 **(b)**

The total number of cases is $11!/2! \times 2!$ The number of favourable cases is $11!/(2! \times 2!) - 9!$ Therefore, required probability is

$$1 - \frac{9! \times 4}{11!} = \frac{53}{55}$$

11 **(a)**

The divisibility of the product of four numbers depends upon the value of the last digit of each number. The last digit of a number can be any one of the 10 digits 0,1, 2, …9. So, the total number of ways of selecting last digits of four numbers is $10 \times 10 \times 10 \times 10 = 10^4$. If the product of the 4 numbers is not divisible by 5 or 10, then the number of choices for the last digit of each number is 8 (excluding 0 or 5). So, favourable number of ways is 8^4. Therefore, the probability that the product is not divisible by 5 or 10 is $(8/10)^4$. Hence, the required probability is $1 - (8/10)^4 = 369/625$

12 **(c)**

Let E = event when each American man is seated adjacent to his wife

and A = event when Indian man is seated adjacent to his wife.

Now, $n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife.

Again, $n(E) = (5!) \times (2!)^4$

$$\therefore P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)}$$

$$= \frac{(4!) \times (2!)^3}{(5!) \times (2!)^4} = \frac{2}{5}$$

13 **(c)**

Let the probability that a man aged x dies in a year p. Thus the probability that a man aged x does not die in a year $= 1 - p$. The probability that all n men aged $x$do not die in a year is $(1 - p)^n$. Therefore, the probability that at least one man dies in a year is $1 - (1 - p)^n$. The probability that out of n men, A_1 dies first is $1/n$. Since this event is independent of the event that at least one man dies in a year, hence, the probability that A_1 dies in the year and he is first one to die is $1/n[1-(1-p)^n]$

14 **(b)**

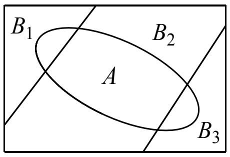
Let us consider the following events

A: card shows up black

B_1: card with both sides black

B_2: card with both sides white

B_1: card with one side white and one black



$P(B_1)=\frac{2}{10}, P(B_2)=\frac{3}{10}, P(B_3)=\frac{5}{10}$

$P(A/B_1)=1,\ P(A/B_2)=0\ \ P(A/B_3)=\frac{1}{2}$

$$P(B_1/A)=\frac{\frac{2}{10}\times 1}{\frac{2}{10}\times 1+\frac{3}{10}\times(0)+\frac{5}{10}\times\frac{1}{2}}=\frac{4}{4+5}=\frac{4}{9}$$

15 **(c)**

Possibilities of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

$\therefore p=\frac{4}{36}=\frac{1}{9}$ and $q=1-\frac{1}{9}=\frac{8}{9}$

Therefore, the required probability is

$^3C_2q^1p^2=(3)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)^2=\frac{8}{243}$

16 **(d)**

The number of ways of arranging n numbers is $n!$ In each order obtained, we must now arrange the digits 1, 2, …k as group and the $n-k$ remaining digits. This can be done in $(n-k+1)!$ ways. Therefore, the probability for the required event is $(n-k+1)!/n!$

17 **(a)**

For each toss, there are four choices:

1. A gets head, B gets head

2. A gets tail, B gets head

3. A gets head, B gets tail

4. A gets tail, B gets tail

Thus, exhaustive number of ways is 4^{50}. Out of the four choices listed above, (iv) is not favourable to the required event in a toss. Therefore, favourable number of cases is 3^{50}. Hence, the required probability is $(3/4)^{50}$

18 **(c)**

Let a_n be the number5 of strings of H and T of length n with no two adjacent H's. Then $a_1 = 2$, a_2 $= 3$. Also,
$a_{n+2} = a_{n+1} + a_n$ (since the string must with T or HT)
So, $a_3 = 5$, $a_4 = 8$, $a_5 = 8 + 5 = 13$
Therefore, the required probability is $13/2^5 = 13/52$
19 **(a)**
We have ratio of the ships A, B and C for arriving safely are 2:5,3:7 and 6:11, respectively.
Therefore, the probability of ship A for arriving safely is 2/(2+5)=2/7
Similarly, for B the probability is 3/(3+7)=3/10 and for C the probability is $C = 6/(6 + 11) = 6/17$
Therefore, the probability of all the ships for arriving safely is $(2/7) \times (3/10) \times (6/17)$ 18/595
20 **(a)**
Out of 9 socks, 2 can be drawn in ${}^9C_2$ ways. Therefore, the total number of cases is ${}^9C_2$. Two socks drawn from the drawer will match if either both are brown or both are blue. Therefore, favourable number of cases is ${}^5C_2 + {}^4C_2$. Hence, the required probability is

$$\frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{4}{9}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	A	D	B	A	C	C	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	B	C	D	A	C	A	A