

Topic :- PROBABILITY

3.
$$\left. \begin{array}{l} np = 4 \\ npq = 2 \end{array} \right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$$

$$P(X = 2) = {}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = 28 \cdot \frac{1}{2^8} = \frac{28}{256}$$

6. Let A be the event that sum of digits is 8
exhaustive cases $\rightarrow {}^{50}C_1$
favourable cases $\rightarrow 08, 17, 26, 35, 44 = {}^5C_1$

$$P(A) = \frac{{}^5C_1}{{}^{50}C_1}$$

Let B be the event that product of digits is zero
favourable cases \rightarrow
 $\{00, 01, \dots, 09, 10, 20, 30, 40\} = {}^{14}C_1$

$$\therefore P(B) = \frac{{}^{14}C_1}{{}^{50}C_1}$$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/{}^{50}C_1}{{}^{14}C_1/{}^{50}C_1} = \frac{1}{14}$$

7. The probability of at least one success

$$1 - \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$$

$$\left(\frac{3}{4}\right)^n \leq \frac{1}{10}$$

$$n \geq \log_{3/4} \left(\frac{1}{10}\right)$$

$$n \geq \frac{-\log 10}{\log_{10} 3 - \log_{10} 4}$$

$$n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$$

8. Required probability = $\frac{{}^3C_1}{{}^9C_1} \cdot \frac{{}^4C_1}{{}^8C_1} \cdot \frac{{}^2C_1}{{}^7C_1} \cdot 3! = \frac{2}{7}$

$$9. P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$$

$$P(D) = P\left(\frac{C}{D}\right) \leq 1$$

$$P(C) \leq P\left(\frac{C}{D}\right)$$

$$P\left(\frac{C}{D}\right) \geq P(C)$$

10. at least one failure = 1 – all success

$$1 \geq 1 - p^5 \geq \frac{31}{32}$$

$$0 \leq p^5 \leq \frac{1}{32}$$

$$0 \leq p \leq \frac{1}{2}$$

$$p \in \left[0, \frac{1}{2}\right]$$

11. $P(A \cap B \cap C) = 0$

$$P\left(\frac{\bar{A} \cap \bar{B}}{C}\right) = \frac{P\{(\bar{A} \cap \bar{B}) \cap C\}}{P(C)} = \frac{P(\bar{A} \cap \bar{B})P(C)}{P(C)}$$

$$= \frac{[1 - P(A) - P(B) + P(A)P(B)]P(C)}{P(C)}$$

($\because P(A \cap B \cap C) = 0$)

$$= \frac{P(C) - P(A)P(C) - P(B)P(C)}{P(C)}$$

$$= 1 - P(A) - P(B) = P(A^c) - P(B)$$

12. Let Events A denotes the getting min No. is

3 & B denotes the max. no. is 6

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$$

Aliter

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{{}^4C_3 - 2}{{}^6C_3 - {}^5C_3} = \frac{2}{{}^8C_3} = \frac{2}{10} = \frac{1}{5}$$

13. $P(4\text{correct}) + P(5\text{ correct})$

$$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^5 = \frac{11}{3^5}$$

$$16. P = {}^{14}C_{13} \frac{1}{2} \left(\frac{1}{2}\right)^{14-13} = 14 \times \frac{1}{2^{13}} \frac{1}{2} = \frac{7}{2^{13}}$$

17. (B)

Since the two boys came out are a girl and a boy, students 3 are boys and 1 is a girl

therefore remaining 2 are boys iff among the 4

$$\therefore \text{probability} = {}^4C_3 \left(\frac{1}{2}\right)^4 = \frac{1}{4}$$

18. A ball from first urn can be drawn in two manners

ball is white or ball is black

$$P(W) = \frac{m}{m+n} \quad P(B) = \frac{n}{m+n}$$

Let E → selecting a white ball from second urn after a ball from urn first has been placed into it

$$\begin{aligned} P(E) &= P(W) P(E/W) + P(B) P(E/B) \\ &= \frac{m}{m+n} \times \frac{p+1}{p+q+1} + \frac{n}{m+n} \times \frac{p}{p+q+1} \\ &= \frac{m(p+1)+np}{(m+n)(p+q+1)} \end{aligned}$$

19. (A)

Total number of functions from A to B = $n(S) = 5^7$

total number of onto functions from A to B is

$$n(E) = \frac{7!}{3!4!} \times 5! + \frac{7!}{3!2!} \times \frac{1}{2!2!} \times 5! = \frac{7! \times 20}{6}$$

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

20. Last place can be occupied by (0 – 9) 10 methods.

to get '6' at unit place of x^4 Last digit should be 2, 4, 6 or 8 is 4 ways

$$\Rightarrow P = \frac{4}{10} = 40\%$$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	B	C	C	C	B	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	C	C	A	B	B	A	D

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