

Topic :- PROBABILITY

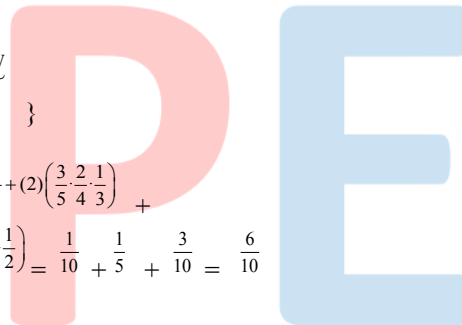
2. There are 6 possible arrangements of a, b, c and only one of them is in alphabetical order.

Alternatively $\frac{{}^6C_3 (\text{for a,b,c}) \cdot 3!}{6!} = \frac{1}{6}$

4. $\left\{ \begin{matrix} WW \\ RRR \end{matrix} \right.$

$S = \{WWW \text{ or } \frac{RWWR}{WRW} \text{ or } \frac{RRWW}{\frac{WRRW}{RWRW}} \}$

$P(\text{last drawn ball is white}) = \frac{2}{5} \cdot \frac{1}{4} + (2) \left(\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \right) +$
 $(3) \left(\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \right) = \frac{1}{10} + \frac{1}{5} + \frac{3}{10} = \frac{6}{10}$



5. $P(H) = p$; $P(T) = 1 - p$

A wins if A throws a Tail before B tosses a Head

$P(A) = P(T \text{ or } HTT \text{ or } HTHTT \text{ or } \dots)$

[HTT \Rightarrow A throws Head and B throws Tail and again A throws a Tail]

$\therefore \frac{1-p}{1-p(1-p)} = \frac{1}{2} \Rightarrow 1-p+p^2 = 2-2p$

$\Rightarrow p^2 + p - 1 = 0$

$\Rightarrow p = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{\sqrt{5}-1}{2}$

7. Determinant = $ad - bc$

probability that randomly chosen product (xy) will be odd

$= P(\text{both odd}) = p^2$

\therefore Probability (xy) is even = $1 - p^2$

Now $(ad - bc)$ is even

\Rightarrow both odd or both even = $p^4 + (1 - p^2)^2$

$$\text{Hence } p^4 + (1 - p^2)^2 = \frac{1}{2}$$

$$\text{put } p^2 = t; t^2 + (1 - t)^2 = \frac{1}{2} \Rightarrow 2t^2 - 2t + 1 = 0 \Rightarrow 4t^2 - 4t + 1 = 0$$

$$(2t - 1)^2 = 0 \Rightarrow t = \frac{1}{2}; \quad \therefore p^2 = \frac{1}{2};$$

$$\text{Hence } p = \frac{1}{\sqrt{2}}$$

9. letters Digits

$$\boxed{}\boxed{}\boxed{} \quad \boxed{}\boxed{}\boxed{}$$

$$26 \times 26 \times 1 \quad 10 \times 10 \times 1$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{26 \times 26 \times 1}{(26)^3} + \frac{10 \times 10 \times 1}{(10)^3} - \frac{26 \times 26 \times 1}{(26)^3} \cdot \frac{10 \times 10 \times 1}{(10)^3}$$

$$= \frac{1}{26} + \frac{1}{10} - \frac{1}{260} = \frac{7}{52}$$

12. $n(S) = 216$

x_1 : number appearing on first dice.

x_2 : number appearing on second dice.

x_3 : number appearing on third dice.

$$x_1 + x_2 + x_3 \leq 10 \quad (x_1, x_2, x_3 \in [1, 6])$$

$$\Rightarrow x_1 + x_2 + x_3 \leq 7 \quad (\text{after giving 1 each to } x_1, x_2, x_3)$$

$$x_1 + x_2 + x_3 + X = 7 \quad (\text{adding } X \text{ as a false beggar})$$

$$\text{Total number of solutions } {}^{(7+3)}C_3 = {}^{10}C_3 = 120$$

Now, number of solutions when any one of x_1, x_2, x_3 takes the value 7 is $x_1 + x_2 + x_3 + X = 1$

$$\Rightarrow {}^{(1+3)}C_3 = {}^4C_3 = 4$$

$$\therefore \text{total number of ways are } {}^{10}C_3 - 4 {}^3C_1$$

$$= 120 - 12 = 108$$

$$\therefore \text{required probability is } \frac{108}{216} = \frac{1}{2}$$

14. Given $P(A) = \frac{1}{16}$; $P(B) = \frac{1}{16}$; $P(C) = \frac{2}{16}$;

$$P(D) = \frac{3}{16}; P(e) = \frac{4}{16}; P(f) = \frac{5}{16};$$

$$P(a, c, e) = P(A) = \frac{1}{16} + \frac{2}{16} + \frac{4}{16} = \frac{7}{16}; A^c = \{b, d, f\}$$

$$P(c, d, e, f) = P(B) = \frac{2}{16} + \frac{3}{16} + \frac{4}{16} + \frac{5}{16} = \frac{8}{16}$$

$$P(b, c, f) = P(C) = \frac{1}{16} + \frac{2}{16} + \frac{5}{16} = \frac{8}{16}$$

$$p_1 = P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(c, e)}{P(B)} = \frac{6}{14} = \frac{3}{7}$$

$$p_2 = P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{P(c, f)}{P(C)} = \frac{7}{8} = \frac{7}{8}$$

$$p_3 = P(C/A^c) = \frac{P(C \cap A^c)}{P(A^c)} = \frac{P(b,f)}{P(b,d,f)} = \frac{6}{9} = \frac{2}{3}$$

$$p_4 = P(A^c/C) = \frac{P(A^c \cap C)}{P(C)} = \frac{P(b,f)}{P(b,c,f)} = \frac{6}{8} = \frac{3}{4}$$

Hence $p_1 = \frac{72}{168}$; $p_2 = \frac{147}{168}$; $p_3 = \frac{112}{168}$; $p_4 = \frac{126}{168}$;
 $p_1 < p_3 < p_4 < p_2 \Rightarrow (C)$

15. Events are defined as

E_1 = A rigged die is chosen

E_2 = A fair die is chosen

A = die shows 5 in all the three times

using Baye's Theorem :

$$P(E_1/A) = \frac{P(E_1) P(A/E_1)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)}$$

$$= \frac{\frac{1}{4} \times (1)^3}{\frac{1}{4} \times (1)^3 + \frac{3}{4} \times \left(\frac{1}{6}\right)^3} = \frac{216}{219}$$

16. 'a' can take only one value i.e. 2 Note: absolute

'b' can be 1 or 3 i.e. two values

'c' can be 2 or 4 i.e. two values

and 'd' can take only one value i.e. 5

hence total favourable ways = $1 \times 2 \times 2 = 4$

$$n(S) = 6^4 = 1296$$

$$P(E) = \frac{4}{1296} = \frac{1}{324}$$

17. Probability problem is not solved by A = $1 - \frac{1}{2} = \frac{1}{2}$

Probability problem is not solved by B = $1 - \frac{1}{3} = \frac{2}{3}$

Probability problem is not solved by C = $1 - \frac{1}{4} = \frac{3}{4}$

Probability of solving the problem = $1 - P$ (not solved by any body)

$$\therefore P = 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

18. $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$

$$P(\bar{A}) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$$

$$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\frac{1}{4} = \frac{1}{3} + P(B) - \frac{3}{4} \Rightarrow P(B) = \frac{2}{3}$$

$$P(\cap B) = P(B) - P(A \cap B)$$

$$= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$$

19. Probability of getting odd $p = \frac{3}{6} = \frac{1}{2}$

Probability of getting others $q = \frac{3}{6} = \frac{1}{2}$

Variance = $npq = 5 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{4}$

20. Out of 5 horses only one is the winning horse. The probability that Mr. A selected the losing horse = $\frac{4}{5} \times \frac{3}{4}$

\therefore The probability that Mr. A selected the winning horse = $1 - \frac{4}{5} \times \frac{3}{4} = \frac{2}{5}$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	C	D	A	D	D	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	C	C	A	A	A	D	D

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