DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-MATRICES

1 **(c)**

It is given that

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & a+b \\ c & c+d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix}$

$\Rightarrow a+c=a, a+b=b+d, c+d=d$

$\Rightarrow c=0$ and $a=d$

2 **(a)**

$AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$

3 **(a)**

Let A be a skew-symmetric matrix of odd order $(2n+1)$ say. Since A is skew-symmetric

$\therefore A^T = -A$

$\Rightarrow |A^T| = |-A|$

$\Rightarrow |A^T| = (-1)^{2n+1}|A|$

$\Rightarrow |A^T| = -|A|$

$\Rightarrow |A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$

4 **(a)**

As, $PP^T = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow PP^T = I$ or $P^T = P^{-1}$ ...(i)

As, $Q = PAP^T$

$\therefore P^TQ^{2005}P = P^T[PAP^T)(PAP)^T)...2005 \text{ times}]P$

$= \underbrace{(P^TP)A(P^TP)A(P^TP)...(P^TP)A(P^T P)}_{2005\ times}$

$= IA^{2005} = A^{2005}$

$\therefore A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$...and so on

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow P^T Q^{2005} P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

5 **(c)**

Given, $2X + 3Y = 0$ ... (i)

and $X + 2Y = I$ ...(ii)

where $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On solving Eqs. (i) and (ii), we get

$X = -3I = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

6 **(d)**

Subtracting the addition of first two equations from third equation, we get

$0 = -5$ which is an absurd result.

7 **(d)**

Given $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$

$|A| = \begin{vmatrix} x & -2 \\ 3 & 7 \end{vmatrix} = 7x + 6$

$\therefore \ A^{-1} = \dfrac{1}{7x+6} \begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$

But given $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$

$\therefore \quad \dfrac{7}{7x+6} = \dfrac{7}{34}$

$\Rightarrow \quad 7x + 6 = 34 \Rightarrow 7x = 28 \Rightarrow x = 4$

8 **(d)**

(a) It is clear that A is not a zero matrix.

(b) $(-1)I = -1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

ie, $\quad (-1)I \neq A$

(c) $|A| = 0 \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} - 0 \begin{vmatrix} 0 & 0 \\ -1 & 0 \end{vmatrix} - 1 \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}$

$= 0 - 0 - 1(-1) = 1$

Since, $|A| \neq 0$ so A^{-1} exists.

(d) $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^2 = I$

10 **(a)**

Since, $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore\ AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+4+0 & 0+2-1 & 0+0-3 \\ 3+0+0 & 0+0+2 & 0+0+6 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{bmatrix}$

$= \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$

11 **(c)**

We have,

$[F(x)\,G(y)]^{-1} = [G(y)]^{-1}\,[f(x)]^{-1}$

$\Rightarrow [F(x)\,G(y)]^{-1} = G(-y)\,F(-x)$

12 **(a)**

$A^{-1} = \dfrac{1}{1+10}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \dfrac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$

Also, $A^{-1} = xA + yI$

$\Rightarrow\ \dfrac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} x & 2x \\ -5x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$

$\Rightarrow\ x + y = \dfrac{1}{11}, 2x = \dfrac{-2}{11}$

$\Rightarrow\ x = \dfrac{-1}{11},\ y = \dfrac{2}{11}$

13 **(d)**

Now, $(AB)^T = B^T A^T$

14 **(c)**

On comparing corresponding elements, we get

$x + y + z = 9$

$x + y = 5$

and $y + z = 7$

On solving these, we get $x = 2, y = 3, z = 4$

$\Rightarrow\ (x,y,z) = (2,3,4)$

15 **(c)**

$A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$= \begin{bmatrix} = a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$

$\Rightarrow\ A^2 = 0$

$\therefore A$ is nilpotent matrix of order 2.

16 **(d)**

Since A,B and C are non-singular matrices, then

$(AB^{-1}C)^{-1} = C^{-1}(AB^{-1})^{-1}$

$= C^{-1}\left((B^{-1})^{-1} A^{-1}\right) = C^{-1}BA^{-1}$

17 **(a)**

Given matrix is invertible

$\Rightarrow \begin{vmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} \neq 0$

$\Rightarrow \lambda\ (0-1) + 1(-6+1) + 4(-3-0) \neq 0$

$\Rightarrow \quad -\lambda - 5 - 12 \neq 0$

$\Rightarrow \lambda \neq -17$

18 **(d)**

From Eqs. (ii) and (iii), we get

$\frac{3y^2}{b^2} - \frac{3z^2}{c^2} = 0$

$\Rightarrow \frac{z^2}{c^2} = \frac{y^2}{b^2}$

On putting this value in Eq. (i), we get

$\frac{2x^2}{a^2} - \frac{2y^2}{b^2} = 0$

$\Rightarrow \frac{x^2}{a62} = \frac{y^2}{b^2}$

$\therefore \frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2} = k^2$ (say)

$\Rightarrow x = \pm ka, y = \pm kb, z = \pm kc, \forall k \in R$

19 **(b)**

We have, $A = \begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$

$\therefore |A| = 1 - \log_a b \log_b a = 1 - 1 = 0$

20 **(a)**

$|A| = 4 - 6 = -2$

adj $(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\therefore A^{-1} = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	A	A	C	D	D	D	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	D	C	C	D	A	D	B	A