DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-MATRICES

2 **(b)**

Let$\Delta = \begin{bmatrix} a_1 & a_2 \\ a_4 & a_5 \end{bmatrix}$

$= a_1a_5 - a_2a_4$

$= a_1(a_1 + 4d) - (a_1 + d)(a_1 + 3d)$

$= a_1^2 + 4a_1d - a_1^2 - 4a_1d - 3d^2 = -3d^2 \neq 0$

Hence, given system of equations has unique solution.

3 **(b)**

$\because \; I_n = \begin{bmatrix} 1 & 0 & 0 \cdots & 0 \\ 0 & 1 & 0 \cdots & 0 \\ 0 & 0 & 1 \cdots & 0 \\ \vdots & \vdots & \vdots \; \vdots & \vdots \\ 0 & 0 & 0 \cdots & 1 \end{bmatrix}, |I_n| = 1$

adj $(I_n) = I_n$

$\therefore \; (I_n)^{-1} = I_n$

4 **(c)**

We know that

$(\text{adj } A)^T = \text{adj } A^T$

$\Rightarrow \text{adj } A^T - (\text{adj } A)^T = O$ (Null matrix)

5 **(c)**

Given, $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 10 \end{bmatrix}$

It is of the form $AX = B$ ...(i)

$|A| = 2(3 + 2) + 1(1 + 3) + 3(2 - 9) = -7$

$\therefore \; \text{adj }(A) = \begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{-7} \begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix}$

From Eq.(i),$X = -\frac{1}{7} \begin{bmatrix} 5 & 7 & -8 \\ -4 & -7 & 5 \\ -7 & -7 & 7 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ 10 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -7 \\ -14 \\ -21 \end{bmatrix}$

$\Rightarrow \quad x = 1, y = 2, z = 3$

7 **(b)**

$A = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 4 & 1 & 1 \end{vmatrix}$

$= 1(1+1) + 1(2+4) + 1(2-4) = 6 \neq 0$

Hence, it has unique solution.

8 **(d)**

Given, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Now, $|A| = \cos^2\theta + \sin^2\theta = 1 \neq 0.$

$\therefore A$ is invertible.

9 **(b)**

$|A| = -1$ and $\text{adj } A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A$

10 **(b)**

$A = \begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & a+6 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix}$

[using $R_1 \to R_1 + R_3$ and $R_2 \to R_2 - 2R_3$]

$= \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix}$ [using $R_1 \to R_1 + R_2$]

When $a = -6$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -2 & -5 \end{vmatrix}$ $\therefore \rho(A) = 1$

When $a = 6$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -12 \\ 1 & -2 & 7 \end{vmatrix}$, $\therefore \rho(A) = 2$

When $a = 1$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -7 \\ 1 & -2 & 2 \end{vmatrix}$, $\therefore \rho(A) = 2$

When $a = 2$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -8 \\ 1 & -2 & 3 \end{vmatrix}$ $\therefore \rho(A) = 2$

11 **(b)**

Let $D = \begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix}$

Then, $|D| = d_1\, d_2 \cdots d_n$

Now, Cofactor of $D_{11} = d_2\, d_3 \cdots d_n$
Cofactor of $D_{22} = d_1\, d_3 \cdots d_n$ etc
And, Cofactor of $D_{ij} = 0$ when $i \neq j$

$$\therefore D^{-1} = \frac{1}{|D|} \text{ adj } D$$

$$= \frac{1}{d_1\, d_2 \cdots d_n}\begin{bmatrix} d_2\, d_3 \cdots d_n & 0 & 0 \cdots & 0 \\ 0 & d_2 d_3 \cdots d_n & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \\ 0 & 0 & 0 \cdots & d_1\, d_2 \cdots d_{n-1} \end{bmatrix}$$

$$\therefore D^{-1} = \begin{bmatrix} \frac{1}{d_1} & 0 & 0 \cdots & 0 \\ 0 & 0 & 0 \cdots & 0 \\ 0 & \vdots & \vdots & \frac{1}{d_n} \\ \vdots & 0 & 0 & \\ 0 & & & \end{bmatrix} = \text{diag}\left(s_1^{-1}\, d_2^{-1} \cdots d_n^{-1}\right)$$

12 **(d)**

$$\because A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$$

$$\therefore f(A) = A^2 + 4A - 5$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

13 **(a)**

$$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

Again now, $4A - 3I = 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$$\therefore \quad A^2 = 4A - 3I$$

14 **(d)**

$$\because (AB)^{-1} = B^{-1}A^{-1}$$

15 **(d)**

$|A| = -8$

$$\text{adj}(A) = \begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ -4 & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-8}\begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ -4 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1/2 \\ 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$$

16 **(b)**

Since given system of equations possesses a non-zero solution.

$$\therefore \quad \Delta = \begin{bmatrix} a & 1 & 1 \\ 1 & -a & 1 \\ 1 & 1 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad a(-a-1) - 1(1-1) + 1(1+a) = 0$$

$\Rightarrow a^2 = 1 \;\Rightarrow\; a = \pm 1$

17 **(a)**

Now, $(A + A^T)^T = A^T + (A^T)^T = A^T + A \quad \therefore A + A^T$ is symmetric matrix.

18 **(c)**

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} = 14$$

$\because$ (adj(adj A)) $= |A|^{n-2}A = 14^{3-2}A = 14\,A$

$\therefore$ |adj (adj A)| $= |14A| = 14^3|A| = 14^4$

19 **(b)**

Given, $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} x + y + z \\ x - 2y - 2z \\ x + 3y + z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$

On Comparing both sides, we get

$x + y + z = 0$ ...(i)

$x - 2y - 2z = 3$ ...(ii)

and $x + 3y + z = 4$ ...(iii)

On solving Eqs. (i), (ii) and (iii) , we get

$x = 1, y = 2$ and $z = -3$

20 **(a)**

$$|A| = \begin{vmatrix} 1 & 2 \\ -4 & -1 \end{vmatrix}$$

$= -1 + 8 = 7$

adj $A = \begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{7}\begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	C	C	A	B	D	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	A	D	D	B	A	C	B	A