DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-MATRICES

2 **(d)**

$X^2 = X \cdot X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

For $n = 2$, no option is satisfied

Hence, option (d) is correct

3 **(a)**

We have,

$F(\alpha)F(-\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow F(\alpha)F(-\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow F(-\alpha) = [F(\alpha)]^{-1}$

4 **(b)**

We have, $(AA^T)^T = (A^T)^T A^T = AA^T$

$\therefore$ AA^T is symmetric matrix

5 **(c)**

For any square matrix X, we have

$X(\text{adj } X) = |X| \, I_n$

Taking $X = \text{adj } A$, we have

$(\text{adj } A)(\text{adj }(\text{adj } A)) = |\text{adj } A| \, I_n$

$\Rightarrow \text{adj } A(\text{adj }(\text{adj } A)) = |A|^{n-1} I_n$ $[\because |\text{adj } A| = |A|^{n-1}]$

$\Rightarrow (A \text{ adj } A)(\text{adj }(\text{adj } A)) = |A|^{n-1} A$ $\quad [\because A \, I_n = A]$

$\Rightarrow (|A| \, I_n)(\text{adj }(\text{adj } A)) = |A|^{n-1} A$

$\Rightarrow \text{adj }(\text{adj } A) = |A|^{n-2} A$

6 **(d)**

Given equations are

$(\alpha + 1)^3 x + (\alpha + 2)^3 y - (\alpha + 3)^3 = 0$

$(\alpha + 1)x + (\alpha + 2)y - (\alpha + 3) = 0$

and $\quad x + y - 1 = 0$

Since, this system of equations is consistent.

$\therefore \begin{vmatrix}(\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+3)^3 \\ (\alpha+1) & (\alpha+2) & -(\alpha+3)\end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1$ *and* $C_3 \to C_3 + C_1$

$$\Rightarrow \begin{bmatrix}(\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+1)^3 \\ & (\alpha+1)^3 - (\alpha+3)^3 & \\ (\alpha+1) & (\alpha+2) & -(\alpha+1) \\ & -(\alpha+3) + (\alpha+1) & \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}(\alpha+1)^3 & 3\alpha^2+9\alpha+7 & -6\alpha^2-24\alpha-26 \\ (\alpha+1) & 1 & -2 \\ 1 & 0 & 0\end{bmatrix} = 0$$

$\Rightarrow \quad -2(3\alpha^2+9\alpha+7)+6\alpha^2+24\alpha+26=0$

$\Rightarrow 6\alpha+12=0 \quad \Rightarrow \alpha=-2$

7 **(a)**

We have,

$A^n = \begin{bmatrix}\cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta\end{bmatrix}$

$\Rightarrow \frac{1}{n}A^n = \begin{bmatrix}\frac{\cos n\theta}{n} & \frac{\sin n\theta}{n} \\ \frac{-\sin n\theta}{n} & \frac{\cos n\theta}{n}\end{bmatrix}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n}A^n = \begin{bmatrix}0 & 0 \\ 0 & 0\end{bmatrix}$

8 **(c)**

We have,

$AB = O$

$\Rightarrow \begin{bmatrix}\cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha\end{bmatrix}\begin{bmatrix}\cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta\end{bmatrix} = O$

$\Rightarrow \begin{bmatrix}\cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \cos\beta\sin\alpha\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta)\end{bmatrix} = O$

$\Rightarrow \cos(\alpha-\beta)=0$

$\Rightarrow \alpha-\beta$ is an odd multiple of $\frac{\pi}{2}$

9 **(c)**

$|A||\text{adj } A| = |A|^n$ for order n

$\Rightarrow DD' = D^n$

10 **(c)**

Given, $\begin{bmatrix}1+\omega & 2\omega \\ -2\omega & -b\end{bmatrix} + \begin{bmatrix}a & -\omega \\ 3\omega & 2\end{bmatrix} = \begin{bmatrix}0 & \omega \\ \omega & 1\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}1+\omega+a & \omega \\ \omega & 2-b\end{bmatrix} = \begin{bmatrix}0 & \omega \\ \omega & 1\end{bmatrix}$

$\Rightarrow \quad 1+\omega+a=0,\ 2-b=1$

$\Rightarrow \quad a=-1-\omega, b=1$

$\therefore \quad a^2+b^2=(-1-\omega)^2+1^2$

$= 1 + \omega^2 + 2\omega + 1^2$

$= 0 + \omega + 1 \qquad (\because \; 1 + \omega + \omega^2 = 0)$

$=1+\omega$

12 **(a)**

Given, $2kx - 2y + 3z = 0, x + ky + 2z = 0, 2x + kz = 0$

For non-trivial solution

$$\begin{vmatrix} 2k & -2 & 3 \\ 1 & k & 2 \\ 2 & 0 & k \end{vmatrix} = 0$$

$\Rightarrow 2k(k^2 - 0) + 2(k - 4) + 3(0 - 2k) = 0$

$\Rightarrow \quad 2k^3 - 4k - 8 = 0$

$\Rightarrow (k - 2)(2k^2 + 4k + 4) = 0$

$\Rightarrow \quad k = 2$

13 **(a)**

$$AB = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 + 0 + 15 & -2 + 2 + 0 \\ 4 + 0 + 0 & -4 + 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$$

14 **(d)**

$$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

$$\therefore \quad A^2 = B \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\Rightarrow \quad \alpha^2 = 1$ and $\alpha + 1 = 5$

Which is not possible at the same time.

$\therefore$ No real values of α exists.

15 **(c)**

We have,

$E(\alpha) E(\beta)$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) \\ -\sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} = E(\alpha + \beta)$$

Hence, option (c) is correct.

17 **(c)**

We have, $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

$$\therefore \; A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}-\frac{1}{2} & \frac{1}{2}-\frac{1}{2} \\ -\frac{1}{2}+\frac{1}{2} & -\frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$\therefore$ Matrix A is nilpotent

18 **(d)**

Since, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

Now, $|A| = 1(0-2) + 1(2-3) + 2(4-0) = 5$

$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$

Now, $A^{-1}B = \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

19 **(a)**

Since, A is a skew-symmetric matrix. Therefore,

$A^T = -A \Rightarrow |A^T| = |-A|$

$\Rightarrow |A| = (-1)^n|A|$

Also, n is odd

$\therefore 2|A| = 0 \Rightarrow |A| = 0$

Thus, $|\text{adj } A| = |A|^2 = 0$

20 **(d)**

Given System of equations are

$x + 3y + 2z = 0$

$3x + y + z = 0$

and $2x - 2y - z = 0$

Now, $\Delta = \begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 1 \\ 2 & -2 & -1 \end{vmatrix}$

$= 1(-1+2) - 3(-3-2) + 2(-6-2)$

$= 1 + 15 - 16$

$= 0$

Since, determinant is zero, then it has infinitely many solutions.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	A	B	C	D	A	C	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	D	C	C	C	D	A	D