DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-MATRICES

1 **(a)**

Given system of equations is $x+2y+3z=1,\ 2x+y+3z=2$ and $5x+5y+9z=5$

Now, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix}$

$=1(9-15)-2(18-15)+3(10-5)$

$=-6-6+15$

$=3 \neq 0$

Hence, it has unique solution

2 **(a)**

Let $\Delta = \begin{vmatrix} 4 & 2 & (1-x) \\ 5 & k & 1 \\ 6 & 3 & (1+x) \end{vmatrix}$

Applying $R_1 \to R_1 + R_3$

$\Rightarrow \quad \Delta = \begin{vmatrix} 10 & 5 & 2 \\ 5 & k & 1 \\ 6 & 3 & 1+x \end{vmatrix}$

Applying $C_1 \to C_1 - 2C_2$

$\Rightarrow \quad \Delta = \begin{vmatrix} 0 & 5 & 2 \\ 5-2k & k & 1 \\ 0 & 3 & 1+x \end{vmatrix}$

$\Rightarrow \quad (5-2k)(5+5x-6)=0$

$\Rightarrow \quad k=\frac{5}{2}, \qquad x=\frac{1}{5}$

4 **(b)**

Since, $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$

5 **(c)**

It is a direct consequence of the definition of rank

6 **(a)**

Now, $A(x)A(y) = (1-x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}(1-y)^{-1}\begin{bmatrix} 1 & -y \\ -y & 1 \end{bmatrix}$

$= [(1+xy)-(x+y)]^{-1}\begin{bmatrix} 1+xy & -(x+y) \\ -(x+y) & 1+xy \end{bmatrix}$

$= \left(1-\frac{x+y}{1+xy}\right)^{-1}\begin{bmatrix} 1 & -\frac{x+y}{1+xy} \\ -\frac{x+y}{1+xy} & 1 \end{bmatrix}$

$= A(z)$

7 **(a)**

$A^2(\alpha) = \begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 2\cos\alpha\sin\alpha \\ -2\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} = \mathrm{A}(2\alpha)$

8 **(a)**

Since, A is symmetric matrix, therefore $A^T = A$

Now, $(A^n)^T = (A^T)^n = A^n$

Hence, A^n is a symmetric matrix.

9 **(a)**

Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\therefore\ |A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$

$= 0\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

$= -(1-9) + 2(1-6) = 8 - 10 = -2$

and $\text{Adj } A = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$= -\frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$

10 **(a)**

We know that

$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Clearly, $\frac{1}{2}(A + A^T)$ is a symmetric matrix and $\frac{1}{2}(A - A^T)$ is a skew-symmetric matrix

Now,

$$\frac{1}{2}(A+A^T)=\frac{1}{0}\left\{\begin{bmatrix}2 & 0 & -3\\ 4 & 3 & 1\\ -5 & 7 & 2\end{bmatrix}+\begin{bmatrix}2 & 4 & -5\\ 0 & 3 & 7\\ -3 & 1 & 2\end{bmatrix}\right\}$$

$$\Rightarrow \frac{1}{2}(A+A^T)=\frac{1}{2}\begin{bmatrix}4 & 4 & -8\\ 4 & 6 & 8\\ -8 & 8 & 4\end{bmatrix}=\begin{bmatrix}2 & 2 & -4\\ 2 & 3 & 4\\ -4 & 4 & 2\end{bmatrix}$$

11 **(c)**

Since, the system of linear equations has a non-zero solution , then

$$\begin{bmatrix}1 & 2a & a\\ 1 & 3b & b\\ 1 & 4c & c\end{bmatrix}=0$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix}1 & 2a & a\\ 0 & 3b-2a & b-a\\ 0 & 4c-2a & c-a\end{vmatrix}=0$$

$\Rightarrow \quad (3b-2a)(c-a)-(4c-2a)(b-a)=0$

$\Rightarrow \quad 3bc-3ba-2ac+2a^2=4bc-2ab-4ac+2a^2$

$\Rightarrow \quad 2ac=bc+ab$

On dividing by abc both sides ,we get

$$\frac{2}{b}=\frac{1}{a}+\frac{1}{c}$$

$\Rightarrow$ a,b,c are in HP.

12 **(c)**

Given system of equations is

$x-y+z=3$

$2x+y-z=2$

and $\quad -3x-2ky+6z=3$

$\therefore$ The given system will have infinite solutions.

$$\therefore \begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -1\\ -3 & -2k & 6\end{vmatrix}=0$$

$\Rightarrow \quad 6k-18=0 \Rightarrow k=3$

13 **(a)**

The product of two orthogonal matrix is an orthogonal matrix

14 **(b)**

Given system of equations can be rewritten as $AX=B$

$$\Rightarrow \begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 3 & 5 & -7\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}8\\ 6\\ 14\end{bmatrix}$$

$\therefore \quad |A|=1(7-10)-1(-7-6)+1(5+3)$

$=-3+13+8=18\neq 0$

$\therefore$ Given system has unique solution.

15 **(a)**

Given, equations $(x+ay=0, az+y=0, ax+z=0)$ has infinite soluations.

$\therefore$ Using Crames's rule, its determinant=0

$\Rightarrow \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} 0$

$\Rightarrow \ 1 + a^3 = 0 \ \Rightarrow \ a = -1$

17 **(a)**

Given that, $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \ F(-\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \ F(\alpha)F(-\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & 0 \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow [F(\alpha)]^{-1} = F(-\alpha)$

18 **(b)**

By using inverse of matrix, we know

$|M^{-1}| = |M|^{-1}$holds true

$(M^T)^{-1} = (M^{-1})^T$ holds true

and $(M^{-1})^{-1} = M$ holds true

but $(M^2)^{-1} = (M^{-1})^{-2}$ not true

19 **(a)**

Since, $A^2 - B^2 = (A - B)(A + B)$

$= A^2 - B^2 + AB - BA$

$\Rightarrow \ AB = BA$

20 **(c)**

Given, $x_1 + 2x_2 + 3x_3 = 0$ ...(i)

$2x_1 + 3x_2 + x_3 = 0$ ...(ii)

$3x_1 + x_2 + 2x_3 = 0$ ...(iii)

$\Rightarrow \quad \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

$= 1(6 - 1) - 2(4 - 3) + 3(2 - 9)$

$= -18$

Then, this is system has the unique solution.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	B	C	A	A	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	A	B	A	A	A	B	A	C