DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-MATRICES

1 **(d)**

Given, $A = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore \quad A^2 = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} 9A$

$\therefore \quad A^4 = A^2.A^2 = 9A.9A = 81.9A = 729A$

2 **(a)**

Now, $\begin{vmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{vmatrix}$

$= 1(\omega^3 - 1) - \omega^2(\omega^4 - \omega) + \omega(\omega^2 - \omega^2)$

$= 1(1 - 1) - \omega^2(\omega - \omega) + 0$

$= 0$

Hence, matrix A is singular

3 **(a)**

Given system of equations are

$x + y + z = 6, x + 2y + 3z = 10$

and $\quad x + 2y + \lambda z = \mu$

The given system of equations has infinite number of solutions, if any tow equations will be same *ie*, the last two equations will be same, if $\lambda = 3, \mu = 10$.

4 **(a)**

Given, $(A + B)(A - B) = A^2 - B^2$

$\Rightarrow \quad A^2 - AB + BA - B^2 = A^2 - B^2$

$\Rightarrow \quad AB = BA$

Now, $(ABA^{-1})^2 = (BAA^{-1})^2 = B^2$

5 **(c)**

Since diagonal elements of a skew –symmetric matrix are all zeros i.e. $a_{ii} = 0$ for all i

$\therefore \text{tr}\,(A) = \sum_{i=1}^{n} a_{ii} = 0$

6 **(c)**

$\because\ P63 = P(I - P) \quad \because P^2 = I - P)$
$= PI - P^2 = PI - (I - P)$
Now, $P^4 = P \cdot P^3$
$\Rightarrow\ P^4 = P(2P - I)$
$\Rightarrow\ P^4 = 2P^2 - P$
$\Rightarrow\ P^4 = 2I - 2P - P$
$\Rightarrow\ P^4 = 2I - 3P$
And $P^5 = P(2I - 3P)$
$\Rightarrow\ P^5 = 2P - 3(I - P)$
$\Rightarrow\ P^5 = 5P - 3I$
Also, $P^6 = P(5P - 3I)$
$\Rightarrow P^6 = 5P^2 - 3P$
$\Rightarrow P^6 = 5(I - P) - 3P$
$\Rightarrow P^6 = 5I - 8P$
So, $n = 6$

Alternate Solution

$\because\ P^n = 5I - 8P$
$= 5(I - P) - 3P$
$= P(5P - 3I)\ (\because\ P^2 = I - P)$
$= P(2P - 3P^2)$
$= P^2(2I - 3P)$
$= P^2[2(I - P) - P]$
$= P^2[2P^2 - P]$
$= P^3[2P - I]$
$= P^4\ [I - P]$
$= P^4 \cdot P^2 = P^6$
$\Rightarrow\ n = 6$

7 **(b)**

$A^2 = A.A = AB.A$
$= A.BA = AB = A$

9 **(d)**

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore\ |A| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$

and adj $A = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

hence, $A^{-1} = \frac{1}{|A|}\text{adj } A = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

so, required element $= A^{-1}_{13} = 7$

10 **(a)**

$\because\ |A| = 1$

and $A^c = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and adj $A = (A^c)' = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore\ A^{-1} = \dfrac{\text{adj } A}{|A|} = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-x)$

11 **(b)**

$\because |A| = 1(0-1) = -1$

$\therefore$ Cofactors of A are

$C_{11} = 0, C_{12} = 0, C_{13} = -1$

$C_{21} = 0, C_{22} = -1, C_{23} = 0$

$C_{31} = -1, C_{32} = 0, C_{33} = 0$

$\therefore\ A^{-1} = \dfrac{1}{-1}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = A$

12 **(b)**

We have,

$A^2 - 5\,I_2 - \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5\,A$

$\therefore k = 5$

14 **(b)**

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $B\begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$ $\quad\therefore\ AX = B \quad\quad \Rightarrow X = A^{-1}B$

Here, $A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 2 & 0 \\ -3 & 0 & 3 \\ 5 & -2 & -3 \end{bmatrix}$

$\therefore\ X = \dfrac{1}{6}\begin{bmatrix} 4 & 2 & 0 \\ -3 & 0 & 3 \\ 5 & -2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$

$= \dfrac{1}{6}\begin{bmatrix} 0+6+0 \\ 0+0+12 \\ 0-6-12 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

Thus, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

15 **(a)**

The given system of equations can be rewritten as matrix from $AX = B$ as

$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, $|A| = 1(6+1) + 1(3+2) + 1(1-4)$
$= 7 + 5 - 3 = 9 \neq 0$
Since, $|A \neq 0|$. So, the given system of equations has only trivial solution. So, there is no non-trivial solution.
16 **(d)**
If matrix has no inverse it means the value of determinant should be zero.
$$\therefore \quad \begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix} = 0$$
If we put $x = 1$, then column Ist and IIIrd are identical.
17 **(b)**
Since, $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix
$$\therefore \begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$$
$\Rightarrow (2+x)(5-2) - 3(-5-2x) + 4(1+x) = 0$
$\Rightarrow 6 + 3x + 15 + 6x + 4 + 4x = 0$
$\Rightarrow \ 13x + 25 = 0 \ \Rightarrow \ x = -\dfrac{25}{13}$
18 **(a)**
We have,
$$A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$$
$$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -4 & 2 \end{bmatrix} \quad \text{Applying } R_2 \to 2R_2 + R_3$$
$$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \quad \text{Applying } C_3 \to C_3 - 2C_2, \ C_4 \to C_4 + C_2$$
$$\Rightarrow A \sim \begin{bmatrix} 2 & 3 & -5 & 7 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Applying } R_2 \leftrightarrow R_3$$
Clearly, $\begin{vmatrix} 2 & 3 \\ 0 & -2 \end{vmatrix} \neq 0$ and every minor of order 3 is zero
Hence, rank of A is 2
19 **(b)**
We have,
$$A^2 = AA = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2\,ab \\ 2\,ab & a^2+b^2 \end{bmatrix}$$
$$\therefore A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$
$\Rightarrow \alpha = a^2 + b^2, \beta^2 = 2\,ab$
20 **(d)**
In a square matrix, the trace of A is defined as the sum of the diagonal elements

Hence, trace of $A = \sum_{i=1}^{n} a_{ii}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	A	A	C	C	B	A	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	D	B	A	D	B	A	B	D