DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth | SOLUTIONS | SUBJECT : MATHS

DATE : | | DPP NO. :4

Topic :-MATRICES

1 **(a)**

Given, $x+4ay+az=0$ ...(i)

$x+3by+bz=0$ ...(ii)

And $x+2cy+cz=0$ ...(iii)

For non-trivial solution

$$\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix}=0$$

Applying $R_2 \to R_2-R_1$, $R_3 \to R_3-R_1$

$$\Rightarrow \begin{vmatrix} 1 & 4a & a \\ 0 & 3b-4a & b-a \\ 0 & 2c-4a & c-a \end{vmatrix}=0$$

$\Rightarrow 1[(3b-4a)(c-a)-2(b-a)(c-2a)]=0$

$\Rightarrow 3bc-3ab-4ac+4a^2-2(bc-2ab-ac+2a^2)=0$

$\Rightarrow \quad bc+ab-2ac=0$

$\Rightarrow \quad ab+bc=2ac$

2 **(d)**

We know that

rank $(A\,B) \leq$ rank(A)

and, rank $(A\,B) \leq$ rank (B)

$\therefore$ rank $(A\,B) \leq \min($rank A, rank $B)$

3 **(c)**

Let $A=\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ and $B=[b_{11}\, b_{12}\, b_{13} \cdots b_{1n}]$ be two non-zero column and row matrices respectively

We have, $A\,B=\begin{bmatrix} a_{11}\,b_{11} & a_{11}\,b_{12} & a_{11}\,b_{13} & \cdots a_{11}\,b_{1\,n} \\ a_{21}\,b_{11} & a_{21}\,b_{12} & a_{21}\,b_{13} & a_{21}\,b_{1\,n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}\,b_{11} & a_{m1}\,b_{12} & a_{m1}\,b_{13} & a_{m1}\,b_{1\,n} \end{bmatrix}$

Since A and B are non-zero matrices. Therefore, the matrix AB will also be a non-zero matrix. The matrix AB will have at least one non-zero element obtained by multiplying corresponding non-zero elements of A and B. All the two-rowed minors of A obviously vanish. But, A is a non-zero matrix. Hence, rank $(A = 1)$

4 **(c)**

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$$

$$= -\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

5 **(a)**

If A is any square matrix, then

$AA^{-1} = I$ and $A^{-1}I = A^{-1}$

Since, $A^2 - A + I = O$

$\Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$

$\Rightarrow (A^{-1}A)A - (A^{-1}A) + A^{-1} = 0$

$\Rightarrow A - 1 + A^{-1} = 0 \Rightarrow A^{-1} = I - A$

6 **(a)**

Since, B is invertible, therefore B^{-1} exists

Now, rank $(A) = \text{rank}[(AB)B^{-1}] \leq \text{rank}(AB)$

But $\text{rank}(AB) \leq \text{rank}(A)$

$\therefore$ rank $(AB) = \text{rank}(A)$

7 **(c)**

Given, $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ of order $n = 2$

$$\therefore |\text{adj}(A)| = |A|^{2-1} = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix} = 10$$

8 **(d)**

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

9 **(b)**

Let A denote the matrix every element of which is unity. Then, all the 2-rowed minors of A obviously vanish. But A is a non-null matrix. Hence, rank of A is 1

10 **(d)**

As $\det(A) = \pm 1, A^{-1}$ exists

and $A^{-1} = \frac{1}{\det(A)}(\text{adj } A) = \pm (\text{adj } A)$

All entries in adj (A) are integers.

$\therefore A^{-1}$ has integer entries.

11 **(c)**

Since, A is invertible

$\therefore\ |A| \neq 0 \Rightarrow \begin{bmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix} \neq 0$

$\Rightarrow 1(1-0) + k(0-k) \neq 0$

$\Rightarrow 1 - k^2 \neq 0 \ \Rightarrow k \neq \pm 1$

12 **(b)**

We have,

$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \frac{1}{1+\tan^2\theta} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \frac{1}{1+\tan^2\theta} \begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \frac{1-\tan^2\theta}{1+\tan^2\theta} & \frac{-2\tan\theta}{1+\tan^2\theta} \\ \frac{2\tan\theta}{1+\tan^2\theta} & \frac{1-\tan^2\theta}{1+\tan^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow a = \cos 2\theta, b = \sin 2\theta$

13 **(c)**

We have,

$x^2 + y^2 + z^2 \neq 0$

$\Rightarrow$ At least one of x,y,z is non-zero

Now,

$x = cy + bz, y = az + cx, z = bx + ay$

$\Rightarrow x - cy - bz = 0$

$cx - y + az = 0$

$bx + zy - z = 0$

As at least one of x,y,z is non-zero. Therefore, the above system of equations has non-trivial solutions

$\therefore \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

14 **(c)**

$A^2 - 4A + 10I = A$

$\Rightarrow \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} - 4 \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} + 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -5 & -3-3k \\ 2+2k & -6+k^2 \end{bmatrix} - \begin{bmatrix} 4 & -12 \\ 8 & 4K \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 9-3k \\ -6+2k & 4+k^2-4K \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

$\Rightarrow 9 - 3k = -3, -6 + 2k = 2$ ...(i)

and $4 + k^2 - 4k = k$

$\Rightarrow k^2 - 5k + 4 = 0 \Rightarrow k = 4,1$ But $k = 1$ is not satisfied the Eq (i).

15 **(a)**

Given, $A^2 = 2A - I$

Now, $A^3 = A^2 \cdot A = 2A^2 = -IA$

$= 2A^2 - A = 2(2A - I) - A$

$= 3A - 2I = 3A - (3-1)I$

$\cdots\ \cdots\ \cdots\ \cdots\ \cdots$

$\cdots\ \cdots\ \cdots\ \cdots\ \cdots$

$A^n = nA - (n-1)I$

16 **(c)**

We have, $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow \ A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$

17 **(b)**

It is given that A is an orthogonal matrix $\therefore A\,A^T = I = A^T A \Rightarrow A^{-1} = A^T$

18 **(a)**

Let $A = IA$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -3 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 - 2R_2$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} A$

Applying $R_2 \to -R_2$ and $R_2 \to R_2 - 2R_3$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2 - 3R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} A$

$\therefore \ A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

19 **(a)**

Given that, $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = 2\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

20 **(b)**

Since the given matrix is symmetric

$$\therefore (A)_{12} = (A)_{21} \Rightarrow x + 2 = 2\,x - 3 \Rightarrow x = 5$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	C	C	A	A	C	D	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	C	A	C	B	A	A	B