

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-MATRICES

2 (a)

Given equations are $x - cy - bz = 0$
 $cx - y + az = 0$ and $bx + ay - z = 0$

For non-zero solution

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

3 (b)

We have,

$$\text{Det}(I_n) = 1 (\neq 0) \Rightarrow \text{rank}(I_n) = n$$

4 (a)

The given matrix A is singular, if

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$$

$$\Rightarrow 56\lambda - 128 - 36\lambda + 48 + 20 = 0$$

$$\Rightarrow 20\lambda = 60$$

$$\Rightarrow \lambda = 3$$

5 (c)

$$\text{Let } B = I + A + A^2 + A^3 + \dots \infty$$

$$\Rightarrow AB = A + A^2 + A^3 + \dots \infty$$

$$\Rightarrow B - AB = I$$

$$\Rightarrow B(I - A) = I$$

$$\Rightarrow B = (I - A)^{-1}$$

$$\Rightarrow B = \begin{bmatrix} 0 & -2 \\ -3 & -3 \end{bmatrix}^{-1} = -\frac{1}{6} \begin{bmatrix} -3 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/3 \\ -1/2 & 0 \end{bmatrix}$$

6 (a)

Since A is non-singular matrix

$$\therefore |A| \neq 0 \Rightarrow \text{rank}(A) = n$$

7 (b)

$$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$$

$$\text{Now, } f(A) = A^2 - 3A + 7$$

$$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - 3 \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$$

$$\therefore f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

8 (a)

The given system of equations will have a unique solution, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$$

9 (a)

$$\text{Given, } 2x + y - z = 7 \quad \dots(i)$$

$$x - 3y + 2z = 1 \quad \dots(ii)$$

$$\text{and } x + 4y - 3z = 5 \quad \dots(iii)$$

From Eqs.(i) and (ii), we get

$$5x - y = 15 \quad \dots(iv)$$

From Eqs. (i) and (iii)

$$5x - y = 16 \quad \dots(v)$$

Eqs. (iv) and (v) shows that they are parallel and solution does not exist.

10 (d)

We have,

$$X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

Clearly for $n = 2$, the matrices in options (a), (b), (c) do not tally with $\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

12 (a)

We have,

$$A = [a_{ij}] \therefore |kA| = k^n |A|$$

13 (b)

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

14 (d)

$$\text{Given that, } A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + 0 & 0 + 0 \\ \alpha + 1 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

Also, $B = A^2$ (given)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

Clearly this is not satisfied by any real value of α

15 **(b)**

We have,

$$A (\text{adj } A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow |A| I = 4 I \quad [\because A(\text{adj } A) = |A| I]$$

$$\Rightarrow |A| = 4$$

$$\Rightarrow |\text{adj } A| = |A|^2 \quad [|\text{adj } A| = |A|^{n-1}]$$

16 **(a)**

$$\text{Given, } A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix}$$

$$\begin{aligned} \text{Similarly, } A^{50} &= \begin{bmatrix} \omega^{50} & 0 \\ 0 & \omega^{50} \end{bmatrix} \\ &= \begin{bmatrix} (\omega^2)^{16} \omega^2 & 0 \\ 0 & (\omega^3)^{16} \omega^2 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

$$= \omega^2 A$$

17 **(a)**

We have,

$$AB = I_3$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow x + y = 0$$

18 **(a)**

$$\text{Let } A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore \text{adj}(A) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

19 **(a)**

$$\begin{aligned}\therefore |A| &= 0 - 1(1 - 9) + 2(1 - 6) \\ &= 8 - 10 = -2 \neq 0\end{aligned}$$

$$\text{adj}(A) = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

20 **(a)**

$$|f(\theta)| = 1(\cos^2\theta + \sin^2\theta) = 1$$

$$\text{Now, adj}\{f(\theta)\} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \{f(\theta)\}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-\theta)$$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	A	C	A	B	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	B	D	B	A	A	A	A	A

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