# DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

**SOLUTIONS**

SUBJECT : MATHS
DPP NO. :2

## Topic :-MATRICES

1 **(a)**

$A^2 = 2A - I$

$\therefore \ A^2A = 2AA - IA$

$= 2A^2 - A = 2(2A - I) - A$

$\Rightarrow A^3 = 3A - 2I$

$\Rightarrow A^3.A = 3AA - 2IA = 3(2A - I) - 2A$

$\Rightarrow A^4 = 4A - 3I$

Similarly, $A^n = nA - (n - I)I$

2 **(d)**

$\det(M_r) = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix} = 2r - 1$

$\sum_{r=1}^{2007} \det(M_r) = 2\sum_{r=1}^{2007} r - 2007$

$= 2 \times \frac{2007 \times 2008}{2} - 2007 = (2007)^2$

3 **(a)**

Let $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}$

$\therefore \ |A| = \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{vmatrix}$

$= 0\begin{vmatrix} 0 & 2 \\ -2 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & 0 \\ 1 & -2 \end{vmatrix}$

$= 0 + 2 - 2 = 0$

$\Rightarrow |A| = 0$

Now, $(\text{adj } A)B = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 4-4+6 \\ 2-2+3 \\ 2-2-3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix} \neq O$

$\therefore$ This system of equation is inconsistent, so it has no solution

5 **(c)**

Given, $D = \text{diag}\,(d_1, d_2, d_3, \ldots, d_n)$

$\Rightarrow\ D^{-1} = \text{diag}(d_1^{-1}, d_2^{-1}, \ldots, d_n^{-1})$

6 **(a)**

We have,

$a = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$ [Using PMI]

$\Rightarrow \frac{1}{n}A^n = \begin{bmatrix} \frac{1}{n} & a \\ 0 & \frac{1}{n} \end{bmatrix} \Rightarrow \lim_{n\to\infty} \frac{1}{2}A^n = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$

7 **(d)**

The given system of equations are

$2x + y - 5 = 0$ ...(i)

$x - 2y + 1 = 0$ ...(ii)

and $2x - 14y - a = 0$ ...(iii)

This system is consistent.

$\therefore \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 1 \\ 2 & -14 & -a \end{bmatrix} = 0$

$\Rightarrow 2(2a + 14) — 1(-a - 2) - 5(-14 + 4) = 0$

$\Rightarrow 4a + 28 + a + 2 + 50 = 0$

$\Rightarrow 5a = -80 \quad \Rightarrow \quad a = -16$

8 **(d)**

The system of given equations has no solution, if $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking common $(\alpha + 2)$ from $C_1$, we get

$(\alpha + 2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\Rightarrow \quad (\alpha + 2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a - 1 & 0 \\ 0 & 0 & a - 1 \end{vmatrix} = 0$

$\Rightarrow \quad (\alpha + 2)(\alpha - 1)^2 = 0$

$\Rightarrow \quad \alpha = 1, -2$

But $\alpha = 1$ makes given three equations same. So, the system of equation have infinite solution. So, answer is $\alpha = -2$ for which the system of equations has no solution

10 **(b)**

Given, $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore \quad A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad x^2 + 1 = 1, x = 0$

$\Rightarrow \quad x = 0$

11 **(a)**

Given that, $A^{-1} = \lambda\,(\text{adj } A)$

On comparing with $A^{-1} = \frac{1}{|A|\text{adj}}A$ we get

$\lambda = \frac{1}{|A|}$

Now, $|A| = \begin{vmatrix} 0 & 3 \\ 2 & 0 \end{vmatrix} = 0 - 6 = -6$

$\Rightarrow \lambda = -\frac{1}{6}$

12 **(d)**

$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14} = |A|$

13 **(d)**

Given equation are $x + y + z = 6, x + 2y + 3z = 10$ and $x + 2y + \lambda z = 10$

Since, it is consistent.

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 3) + 1(2 - 2) = 0$

$\Rightarrow \quad \lambda - 3 = 0 \Rightarrow \lambda = 3$

14 **(b)**

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$

$\therefore \quad A^4 = 2A.2A = 4A^2 = 4 \times 2A = 2^3 A$

Similarly, $\quad A^8 = 2^7 A$

$\Rightarrow \quad A^{100} = 2^{99} A$

15 **(d)**

Let $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$\therefore |A| = \cos^2 2\theta + \sin^2 2\theta = 1$

and adj $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{1}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

18 **(a)**

Give equation can be written as,

$2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = 2\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow \quad X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

19 **(c)**

We have,

$A\,B = 0$

$\Rightarrow |A\,B| = 0$

$\Rightarrow |A|\;|B| = 0$

$\Rightarrow |A| = 0$ or $|B| = 0$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Then, $A\,B = O$. But $A \neq O, B \neq O$

20 **(b)**

Given, $A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 1 \\ 4 & -1 & -2 \end{bmatrix}$

$\therefore \; A^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ 8 & 6 & -5 \\ -6 & -3 & 3 \end{bmatrix}$

Now, $A^{-1}D = \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ 8 & 6 & -5 \\ -6 & -3 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 8 \\ -1 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8/3 \\ -1/3 \\ 0 \end{bmatrix}$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | D | A | A | C | A | D | D | B | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | A | D | D | B | D | C | A | A | C | B |
| | | | | | | | | | | |