DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth

DATE :

SOLUTIONS

SUBJECT : MATHS

DPP NO. :10

Topic :-MATRICES

1 **(b)**

Since, $P=\begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q=\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$\therefore\ PQ=\begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$=\begin{bmatrix} -i^2-i^2 & i^2+i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix}$

$=\begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}=\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$

2 **(d)**

$B=\text{adj}(A)=\begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$

Therefore, $\text{adj}(B)=\begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix}$

Now, $|\text{adj } B|=\begin{vmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{vmatrix}=625$

and $|C|=125|A|=125\begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 5 & 0 \end{vmatrix}=625$

$\therefore \frac{|\text{adj}(B)|}{|C|}=\frac{625}{625}=1$

Alternate

$|A|=1(0+3)+1(0+6)+(0-4)$

Now, $\text{adj } B=\text{adj}(\text{adj } A)$

$=|A|A=5A$

$\therefore\ \frac{|\text{adj } B|}{|C|}=\frac{|5A|}{|5A|}=1$

3 **(d)**

Given, $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$\therefore \quad A^{-1}=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}=A^T$

4 **(b)**

The given system of equations posses non-zero solutions,

$\therefore \begin{bmatrix}1 & 1 & 1\\ 1 & a & a\\ 1 & -a & 1\end{bmatrix}=0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix}1 & 1 & 1\\ 0 & a-1 & a-1\\ 0 & -a-1 & 0\end{vmatrix}=0$

$\Rightarrow \quad 1(0-(a^2-1))=0$

$\Rightarrow a^2=1 \Rightarrow \ a=\pm 1$

5 **(a)**

Given, $x\begin{bmatrix}-3\\ 4\end{bmatrix}+y\begin{bmatrix}4\\ 3\end{bmatrix}=\begin{bmatrix}10\\ -5\end{bmatrix}$

$\therefore -3x+4y=10 \qquad$ ...(i)

and $\qquad 4x+3y=-5 \qquad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x=-2, y=1$

7 **(a)**

$|A|=5+6=11$

and $\quad \text{adj } A=\begin{bmatrix}1 & 2\\ -3 & 5\end{bmatrix}$

$A^{-1}=\frac{1}{11}\begin{bmatrix}1 & 2\\ -3 & 5\end{bmatrix}$

8 **(c)**

We know that, if

$A^n=\begin{bmatrix}d_1 & 0 & 0\\ 0 & d_2 & 0\\ 0 & 0 & d_3\end{bmatrix}=\text{diag } [d_1\, d_2\, d_3]$

Then,

$A^n=\begin{bmatrix}d_1^n & 0 & 0\\ 0 & d_2^n & 0\\ 0 & 0 & d_3^n\end{bmatrix}=\text{diag } [d_1^n\, d_2^n\, d_3^n]$

$\therefore A^5=\begin{bmatrix}2^5 & 0 & 0\\ 0 & 2^5 & 0\\ 0 & 0 & 2^5\end{bmatrix}=16\, A$

9 **(c)**

$AB=I \Rightarrow \ B=A^{-1}$

$=\frac{1}{1+\tan^2\theta}\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}$

$= \dfrac{1}{\sec^2\theta}\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix}$

$\Rightarrow \ (\sec^2\theta)\,\mathrm{B} = \begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix} = \mathrm{A}(-\theta)$

10 **(b)**

It is given that $A = [a_{ij}]$ is a skew-symmetric matrix

$a_{ij} = -\,a_{ji}$ for all i, j

$\Rightarrow a_{ii} = -\,a_{ii}$ for all i

$\Rightarrow 2\,a_{ii} = 0$ for all $i \Rightarrow a_{ii} = 0$ for all i

11 **(c)**

We know that, if $A = \text{diag.}\,(d_1\, d_2, \ldots\ldots, d_n)$ is a diagonal matrix, then for any $k \in N$

$A^k = \text{diag}\,(d_1^k, d_2^k, \ldots, d_n^k)$

Here, $A = \text{diag.}\,(a, a, a)$

$\therefore A^n = \text{diag}\left(a^n, a^n, a^n\right) = \begin{bmatrix}a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n\end{bmatrix}$

12 **(b)**

We have,

$(A\,B - B\,A)^T = (A\,B)^T - (B\,A)^T = B^T A^T - A^T B^T$

$\Rightarrow (A\,B - B\,A)^T = B\,A - A\,B \qquad [\because A^T = A, B^T = B]$

$\Rightarrow (A\,B - B\,A)^T = -(A\,B - B\,A)$

So, $A\,B - B\,A$ is skew-symmetric matrix

13 **(b)**

Since $A\,B$ exists

$\therefore$ No. of rows in $B =$ No. of columns in A

$\Rightarrow$ No. of rows in $B = n$

Also, $B\,A$ exists

$\Rightarrow$ No. of columns in $B =$ No. of rows in A

$\Rightarrow$ No. of columns in $= m$

Hence, B is of order $n \times m$

14 **(c)**

We have,

$(k\,A)(\text{adj}\,k\,A) = |k\,A|\,I_n$

$\Rightarrow k(A\,\text{adj}\,k\,A) = k^n|A|\,I_n \qquad [\because |k\,A| = k^n|A|]$

$\Rightarrow A(\text{adj}\,k\,A) = k^{n-1}|A|\,I_n$

$\Rightarrow A\,\text{adj}\,(k\,A) = k^{n-1}A(\text{adj}\,A) \ \ [\because A\,\text{adj}\,A = |A|\,I_n]$

$\Rightarrow A\,\text{adj}\,(k\,A) = A(k^{n-1}\text{adj A})$

$\Rightarrow A^{-1}(A\,\text{adj}\,(k\,A)) = A^{-1}\left(A\left(k^{n-1}\text{adj}\,A\right)\right)$

$\Rightarrow (A^{-1}A)(\text{adj}\,k\,A) = (A^{-1}\,A)(k^{n-1}\,\text{adj}\,A)$

$\Rightarrow I(\text{adj}\,k\,A) = I(k^{n-1}\,\text{adj}\,A)$

$\Rightarrow \text{adj}\,k\,A = k^{n-1}\,(\text{adj}\,A)$

15 **(c)**

It has a non- zero solution if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad -6k + 6 = 0$

$\Rightarrow \quad k = 1$

16 **(b)**

$(aI + bA)^2 = (aI + bA)(aI + bA)$

$\quad = a^2I^2 + aI(bA) + bA(aI) + (bA)^2$

Now, $I^2 = I$ and $IA = A$

$\therefore (aI + bA)^2 = a^2I + 2abA + b^2(A^2)$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore (aI + bA)^2 = a^2I + 2abA$

17 **(b)**

Since A is orthogonal matrix

$\therefore A\,A^T = I = A^T\,A$

$\Rightarrow |A\,A^T| = |I| = |A^T\,A|$

$\Rightarrow |A||A^T| = 1 = |A^T||A|$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

18 **(b)**

Since, given system of equations has no solution, $\Delta = 0$ and any one amongst $\Delta x, \Delta y, \Delta z$ is non-zero.

Where $\quad \Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

And $\Delta z = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -4 \\ 1 & 1 & \lambda \end{vmatrix} = 6 \neq 0$

$\Rightarrow \quad \lambda = 1$

19 **(c)**

Since, A is an idempotent matrix, therefore $A^2 = A$

$$\Rightarrow \begin{bmatrix} 2 & -2 & -16-4x \\ -1 & 3 & 16+4x \\ 4+x & -8-2x & -12+x^2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$$

On comparing, $16 + 4x = 4$

$\Rightarrow \; x = -3$

20 **(a)**

We have,

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and adj $A = \begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & 1 \end{bmatrix}$

Clearly, $|A| = 6 - 8 + 4 = 2$

$\therefore A\,(\text{adj}\,A) = |A|\,I$

$$\Rightarrow\begin{bmatrix}1 & 2 & 2\\ 2 & 3 & 0\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}6 & -2 & -6\\ -4 & 2 & x\\ y & -1 & -1\end{bmatrix}=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2y-2 & 0 & 2x-18\\ 0 & 2 & 3x-12\\ 2y-4 & 0 & x-2\end{bmatrix}=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$$

$\Rightarrow 2y-2=2,\ 2y-4=0,\ 2x-8=0,\ 3x-12=0, x-2=2$

$\Rightarrow x=4,\ y=2 \Rightarrow x+y=6$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	D	B	A	D	A	C	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	B	C	C	B	B	B	C	A