DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-MATRICES

2 **(b)**

$$\left[\frac{1}{2}(A-A')\right]' = \frac{1}{2}(A-A')' = \frac{1}{2}(A'-A)$$

$$= -\frac{1}{2}(A-A')$$

Hence, it is a skew-symmetric matrix.

3 **(b)**

$$\because \text{ adj } A = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

and $\quad |A| = \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 10$

$$\therefore \quad A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

5 **(b)**

Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$$\therefore \text{ adj } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

6 **(d)**

We have,

$3\,A^3 + 2\,A^2 + 5\,A + I = 0$

$\Rightarrow I = -3\,A^3 - 2\,A^2 - 5\,A$

$\Rightarrow I\,A^{-1} = \left(-3\,A^3 - 2\,A^2 - 5\,A\right)A^{-1}$

$\Rightarrow A^{-1} = -3\,A^2 - 2\,A - 5\,I$

8 **(b)**

The given system of equations are

$x + y + z = 0,$

$2x + 3y + z = 0$ and $\quad x + 2y = 0$

Here, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 1(0-2) - 1(0-1) + (4-3)$

$= -2 + 1 + 1 = 0$

$\therefore$ This system has infinite solutions

9 **(c)**

$2X - \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 4 & 4 \\ 7 & 2 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 2 & 2 \\ 7/2 & 1 \end{bmatrix}$

11 **(b)**

Given, $A = A', B = B'$

Now, $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

$\therefore AB - BA$ is a skew-symmetric matrix.

13 **(a)**

Given that, p is a non-singular matrix such that

$1 + p + p^2 + \dots + p^n = O$

$\Rightarrow (1+p)(1 + p + p^2 + \dots + p^n) = O$

$\Rightarrow 1 - p^{n+1} = O$

$\Rightarrow p^{n+1} = 1$

$\Rightarrow p^n \times p^1 = 1$

$\Rightarrow p^n = 1/p$

$\therefore p^{-1} = p^n$

14 **(a)**

Given, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$

$= \frac{1}{40} \begin{bmatrix} 25 + 0 - 25 \\ -25 + 0 + 65 \\ 50 + 0 + 30 \end{bmatrix}$

$= \frac{1}{40} \begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow x = 0, y = 1, z = 2$

$\therefore x + y + z = 0 + 1 + 2 = 3$

15 **(d)**

$\because \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$

Then, $X = y$ and $Y = x$

$ie, \;\; y = x$

16 **(c)**

Given $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} . \dfrac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \dfrac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \dfrac{1-\tan^2\theta}{1+\tan^2\theta} & \dfrac{2\tan\theta}{1+\tan^2\theta} \\ \dfrac{2\tan\theta}{1+\tan^2\theta} & \dfrac{1-\tan^2\theta}{1+\tan^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \;\; a = \cos 2\theta, \;\; b = \sin 2\theta$

17 **(d)**

Given, $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$\therefore \; B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}$

$\Rightarrow \text{adj } A = (B)' = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

$= 3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = 3A'$

18 **(d)** Given equations are

$px + y + z = 0, x + qy + z = 0, x + y + rz = 0$

Since, the system have a non-zero solution, then

$\begin{bmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{bmatrix} = 0$

Applying $C_2 \to C_2 - C_1$and $C_3 \to C_3 - C_2$

$\Rightarrow \begin{vmatrix} p & 1-p & 0 \\ 1 & q-1 & 1-q \\ 1 & 0 & r-1 \end{vmatrix} = 0$

$$\Rightarrow(1-p)(1-q)(1-r)\begin{vmatrix}\frac{p}{1-p} & 1 & 0\\ \frac{1}{1-q} & -1 & 1\\ \frac{1}{1-r} & 0 & -1\end{vmatrix}=0$$

$$\Rightarrow(1-p)(1-q)(1-r)$$
$$\left[\frac{p}{1-p}(1)-1\left(-\frac{1}{1-q}-\frac{1}{1-r}\right)\right]=0$$

Since, $p,q,r \neq 1$

$$\therefore \frac{p}{1-p}+\frac{1}{1-q}+\frac{1}{1-r}=0$$

$$\Rightarrow \quad \frac{1}{1-p}-1+\frac{1}{1-q}+\frac{1}{1-r}=0$$

$$\Rightarrow \frac{1}{1-p}+\frac{1}{1-q}+\frac{1}{1-r}=0$$

19 **(b)** Given, $AB=I \Rightarrow B=A^{-1}$

Now, $A^{-1}=\frac{\text{adj } A}{|A|}$

$$=\frac{\begin{bmatrix}1 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 1\end{bmatrix}}{1+\tan^2\frac{\theta}{2}}$$

$$=\frac{A^T}{\sec^2\frac{\theta}{2}}=\cos^2\frac{\theta}{2}A^T$$

20 **(d)**

Given equations are $3x+y+2z=3$ ...(i) $2x-3y-z=-3$ ...(ii)

and $x+2y+z=4$ ...(iii)

Let $A=\begin{bmatrix}3 & 1 & 2\\ 2 & -3 & -1\\ 1 & 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}3\\ -3\\ 4\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

$$\therefore \ |A|=\begin{vmatrix}3 & 1 & 2\\ 2 & -3 & -1\\ 1 & 2 & 1\end{vmatrix}$$

$$=3\begin{vmatrix}-3 & -1\\ 2 & 1\end{vmatrix}-1\begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix}+2\begin{vmatrix}2 & -3\\ 1 & 2\end{vmatrix}$$

$$=3(-3+2)-1(2+1)+2(4+3)$$

$$=-3-3+14=8$$

$$\text{adj. } A=\begin{bmatrix}-1 & -3 & 7\\ 3 & 1 & -5\\ 5 & 7 & -11\end{bmatrix}^T$$

$$=\begin{vmatrix}-1 & 3 & 5\\ -3 & 1 & 7\\ 7 & -5 & -11\end{vmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}\text{adj }A=\frac{1}{8}\begin{bmatrix}-1 & 3 & 5\\ -3 & 1 & 7\\ 7 & -5 & -11\end{bmatrix}$$

Now, $X=A^{-1}B$

$$=\frac{1}{8}\begin{bmatrix}-1 & 3 & 5\\ -3 & 1 & 7\\ 7 & -5 & -11\end{bmatrix}\begin{bmatrix}3\\ -3\\ 4\end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}-3-9+20\\ -9-3+28\\ 21+15-44\end{bmatrix}=\frac{1}{8}\begin{bmatrix}8\\ 16\\ -8\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$$

$\Rightarrow\ x=1, y=2, z=-1$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	B	D	B	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	A	A	D	C	D	D	B	D