DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth | **SOLUTIONS** | **SUBJECT : MATHS**
DATE : | | **DPP NO. :9**

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(d)**

$$\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{\frac{6\tan x}{1+\tan^2 x}}{5+\frac{3(1-\tan^2 x)}{1+\tan^2 x}}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{6\tan x}{8+2\tan^2 x}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\tan x}{4+\tan^2 x}\right)$$

$$=\tan^{-1}\left(\frac{\frac{\tan x}{4}+\frac{3\tan x}{4+\tan^2 x}}{1-\frac{3\tan^2 x}{4(4+\tan^2 x)}}\right)\left(\text{as}\left|\frac{\tan x}{4}\cdot\frac{3\tan x}{4\tan^2 x}\right|<1\right)$$

$$=\tan^{-1}\left(\frac{16\tan x+\tan^3 x}{16+\tan^2 x}\right)$$

$$=\tan^{-1}(\tan x)=x$$

2 **(a)**

Let $\sin^{-1}x=\theta$. Then, $x=\sin\theta$

Also,

$-\frac{1}{2}\le x\le\frac{1}{2}\Rightarrow-\frac{1}{2}\le\sin\theta\le\frac{1}{2}\Rightarrow-\frac{\pi}{6}\le\theta\le\frac{\pi}{6}$

Now,

$\sin^{-1}(3x-4x^3)$

$=\sin^{-1}(\sin 3\theta)$

$=3\theta\ \left[\because-\frac{\pi}{6}\le\theta\le\frac{\pi}{6}\Rightarrow-\frac{\pi}{2}\le 3\theta\le\frac{\pi}{2}\right]$

$= 3\sin^{-1}x$

3 **(c)**

We have,

$$\tan\theta + \tan\left(\frac{\pi}{3}+\theta\right) + \tan\left(\frac{-\pi}{3}+\theta\right) = K\tan 3\theta$$

$$\Rightarrow \tan\theta + \tan(60+\theta) + \tan(-60+\theta) = K\tan 3\theta$$

$$\Rightarrow \tan\theta + \frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\tan\theta} + \frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta} = K\tan 3\theta$$

$$\Rightarrow \tan\theta + \frac{8\tan\theta}{1-3\tan^2\theta} = K\tan 3\theta$$

$$\Rightarrow \frac{3(3\tan\theta-\tan^3\theta)}{(1-3\tan^2\theta)} = K\tan 3\theta$$

$\Rightarrow 3\tan 3\theta = K\tan 3\theta \Rightarrow K = 3$

4 **(d)**

Let $\cos^{-1}x = \theta$. Then, $x = \cos\theta$

Also,

$$-1 \le x \le 0 \Rightarrow -1 \le \cos\theta \le 0 \Rightarrow \frac{\pi}{2} \le \theta \le \pi$$

Now,

$\cos^{-1}(2x^2-1) = \cos^{-1}(\cos 2\theta) = \cos^{-1}(2\pi - 2\theta)$

$$\Rightarrow \cos^{-1}(2x^2-1) = 2\pi - 2\theta\left[\begin{array}{c}\because \frac{\pi}{2} \le \theta \le \pi \Rightarrow \pi \le 2\theta \le 2\pi \\ \Rightarrow 0 \le 2\pi - 2\theta \le \pi\end{array}\right]$$

$\Rightarrow \cos^{-1}(2x^2-1) = 2\pi - 2\cos^{-1}x$

5 **(c)**

We have,

$\alpha + \beta = \pi$

Also,

$$\alpha = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3}$$

$$\Rightarrow \alpha = \frac{\pi}{3} + \sin^{-1}\frac{1}{3}$$

$$\Rightarrow \alpha < \frac{\pi}{3} + \sin^{-1}\frac{1}{2} \quad [\because \sin^{-1}x \text{ is increasing on } [-1, 1]]$$

$$\Rightarrow \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$$

$\therefore \alpha + \beta = \pi \Rightarrow \beta > \frac{\pi}{2}$. Thus, $\alpha < \beta$

6 **(a)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$$-1 \le x \le 1 \Rightarrow -1 \le \tan\theta \le 1 \Rightarrow -\frac{\pi}{4} \le \theta \le \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$$

Now,

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta \quad \left[\because -\frac{\pi}{4} \le \theta \le \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}\right]$

$= 2\tan^{-1} x$

7 **(a)**

$\sin\left[3\sin^{-1}\left(\frac{1}{5}\right)\right] = \sin\left[\sin^{-1}\left\{3\left(\frac{1}{5}\right) - 4\left(\frac{1}{5}\right)^3\right\}\right]$

$$= \frac{3}{5} - \frac{4}{125} = \frac{71}{125}$$

8 **(a)**

Since, $-\frac{\pi}{2} < \sin^{-1} x \le \frac{\pi}{2}$

$$\therefore \sin^{-1} x_i = \frac{\pi}{2}, 1 \le i \le 20$$

$$\Rightarrow x_i = 1, 1 \le i \le 20$$

Thus, $\sum_{i=1}^{20} x_i = 20$

9 **(d)**

Given, $\sin[\cot^{-1}(1+x)] = \cos(\tan^{-1} x)$

$\therefore \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(1+x^2)}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$\Rightarrow \frac{1}{\sqrt{1+(1+x^2)}} = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow 1 + x^2 + 2x + 1 = x^2 + 1$

$\Rightarrow \qquad x = -\frac{1}{2}$

10 **(b)**

$\therefore \tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$

$= \tan\left[\frac{\pi}{4} + \phi\right] + \tan\left[\frac{\pi}{4} - \phi\right]$

$\left[\text{put } \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right) = \phi \Rightarrow \cos 2\phi = \frac{a}{b}\right]$

$= \frac{1+\tan\phi}{1-\tan\phi} + \frac{1-\tan\phi}{1+\tan\phi}$

$= \frac{2(1+\tan^2\phi)}{1-\tan^2\phi}$

$= \frac{2}{\cos 2\phi} = \frac{2b}{a}$

11 **(b)**

$$\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$$

$$= \tan^{-1}\frac{x}{y} - \tan^{-1}\left[\frac{1-\frac{y}{x}}{1+\frac{y}{x}}\right]$$

$$= \tan^{-1}\frac{x}{y} - \tan^{-1}1 + \tan^{-1}\frac{y}{x}$$

$$= \tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y} - \tan^{-1}1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

12 **(c)**

Here, $x^2 - 2x + 2 = (x-1)^2 + 1 \geq 1$

But $-1 \leq (x^2 - 2x + 2) \leq 1$

Which is possible only when

$$x^2 - 2x + 2 = 1$$

$$\Rightarrow x = 1$$

Then, $a(1)^2 + \sin^{-1}(1) + \cos^{-1}(1) = 0$

$$\Rightarrow a + \frac{\pi}{2} + 0 = 0$$

$$\Rightarrow a = -\frac{\pi}{2}$$

13 **(d)**

$\cos^{-1}\left(-\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right) + 3\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 4\tan^{-1}(-1)$

$$= \pi - \cos^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{6}\right) + 3\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$$
$$+ 4\tan^{-1}(1)$$

$$= \pi - \frac{\pi}{3} - \frac{\pi}{3} + 3\left(\pi - \frac{\pi}{4}\right) + 4.\frac{\pi}{4}$$

$$= \frac{\pi}{3} + 3.\frac{3\pi}{4} + \pi = \frac{43\pi}{12}$$

14 **(b)**

$\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \qquad \theta = \cot^{-1}x$

Since, $1 \leq x < \infty$, therefore $0 \leq \theta \leq \frac{\pi}{4}$

15 **(b)**

Given, $4\sin^{-1}x + \cos^{-1}x = \pi$

$\Rightarrow 4\sin^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$

$\Rightarrow \qquad 3\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \qquad \sin^{-1}x = \frac{\pi}{6}$

$\Rightarrow \qquad x = \frac{1}{2}$

16 **(d)**

$$\cos\left(\frac{33\pi}{5}\right) = \cos\left(6\pi + \frac{3\pi}{5}\right) = \cos\frac{3\pi}{5}$$
$$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right) = \sin\left(-\frac{\pi}{10}\right)$$
$$= \sin^{-1}\sin\left(-\frac{\pi}{10}\right) = -\frac{\pi}{10}$$

17 **(b)**

Given expression

$$= \tan\left[\tan^{-1}\frac{a_2 - a_1}{1 + a_1 a_2} + \tan^{-1}\frac{a_3 - a_2}{1 + a_2 a_3} + \dots + \tan^{-1}\frac{a_n - a_{n-1}}{1 + a_n a_{n-1}}\right]$$

$$= \tan[\tan^{-1} a_2 - \tan^{-1} a_1 + \tan^{-1} a_3 - \tan^{-1} a_2 + \dots + \tan^{-1} a_n - \tan^{-1} a_{n-1}]$$

$$= \tan[\tan^{-1} a_n - \tan^{-1} a_1] = \frac{a_n - a_1}{1 + a_1 a_n}$$
$$= \frac{(n-1)d}{1 + a_1 a_n}$$

18 **(a)**

$$\because \tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$$
$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}}\right) = \frac{\pi}{2}$$
$$\Rightarrow \frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}} = \tan\frac{\pi}{2} \Rightarrow 1 - \frac{ab}{x^2} = 0$$
$$\Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$$

20 **(b)**

Given, $\tan\left\{\sec^{-1}\left(\frac{1}{x}\right)\right\} = \sin(\tan^{-1} 2)$

$\Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1 - x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{1 + 2^2}}\right)$

$\left[\because \tan^{-1} x = \sin^{-1}\frac{x}{\sqrt{1 + x^2}}\right]$

$\Rightarrow \qquad \frac{\sqrt{1 - x^2}}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow \qquad 4x^2 = 5(1 - x^2)$

$\Rightarrow \qquad x^2 = \frac{5}{9} \quad \Rightarrow x = \frac{\sqrt{5}}{3}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	C	D	C	A	A	A	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	D	B	B	D	B	A	C	B