DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :8

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(a)**

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$

Let $s^2 = \frac{a+b+c}{abc}$

Hence, $\theta = \tan^{-1}\sqrt{a^2s^2} + \tan^{-1}\sqrt{b^2s^2} + \tan^{-1}\sqrt{c^2s^2}$

$$= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs)$$

$$= \tan^{-1}\left[\frac{as+bs+cs-abcs^3}{1-abs^2-acs^2-bcs^2}\right]$$

Hence, $\tan\theta = \left[\frac{s[a+b+c)-abcs^2]}{1-(ab+bc+ca)s^2}\right]$

$$= \left[\frac{s[(a+b+c)-(a+b+c)]}{1-s^2(ab+bc+ca)}\right] = 0$$

2 **(a)**

Given, $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$\therefore \quad \frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$

3 **(c)**

$$\tan^{-1}\left(\frac{c_1 - y}{c_1y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2c_1}\right) + \tan^{-1}\left(\frac{c_3 - c_2}{1 + c_3c_2}\right) + \dots + \tan^{-1}\frac{1}{c_n}$$

$$= \tan^{-1}\left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y}\cdot\frac{1}{c_1}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1c_2}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2c_3}}\right) + \dots + \tan^{-1}\frac{1}{c_n}$$

$$= \tan^{-1}\frac{x}{y} - \tan^{-1}\frac{1}{c_1} + \tan^{-1}\frac{1}{c_1} - \tan^{-1}\frac{1}{c_2} + \tan^{-1}\frac{1}{c\_2}$$

$$- \tan^{-1}\frac{1}{c_3} + \dots + \tan^{-1}\frac{1}{c_{n-1}} - \tan^{-1}\frac{1}{c_n} + \tan^{-1}\frac{1}{c_n}$$
$$= \tan^{-1}\left(\frac{x}{y}\right)$$

4 **(d)**

We have,

$\cos\{\tan^{-1}(\tan 2)\}$

$= \cos\{\tan^{-1}(\tan(2-\pi))\} = \cos(2-\pi) = \cos(\pi-2) = -\cos 2$

5 **(c)**

We have, $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x+2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x+2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$$
$$\Rightarrow \left[\frac{2x(x+2)}{x^2+4+4x-x^2+1}\right] = \tan\frac{\pi}{4}$$
$$\Rightarrow \frac{2x(x+2)}{4x+5} = 1$$
$$\Rightarrow 2x^2+4x = 4x+5$$
$$\Rightarrow x = \pm\sqrt{\frac{5}{2}}$$

6 **(a)**

Given series can be rewritten as

$$\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)$$

Now, $\tan^{-1}\left(\frac{1}{1+r+r^2}\right)$

$$= \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$$
$$= \tan^{-1}(r+1) - \tan^{-1}(r)$$
$$\therefore \sum_{r=1}^{n}[\tan^{-1}(r+1) - \tan^{-1}r]$$
$$= \tan^{-1}(n+1) - \tan^{-1}(1)$$
$$= \tan^{-1}(n+1) - \frac{\pi}{4}$$
$$\Rightarrow \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

7 **(c)**

Here, $x^2 - 2x + 2 = (x-1)^2 + 1 \geq 1$

But $-1 \leq (x^2 - 2x + 2) \leq 1$

Which is possible only when

$$x^2 - 2x + 2 = 1$$
$$\Rightarrow x = 1$$

Then, $a(1)^2 + \sin^{-1}(1) + \cos^{-1}(1) = 0$

$$\Rightarrow a + \frac{\pi}{2} + 0 = 0$$
$$\Rightarrow a = -\frac{\pi}{2}$$

8 **(c)**

Given that, $\tan^{-1} x - \tan^{-1} y = \tan^{-1} A$

$$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1} A$$

Hence, $A = \frac{x-y}{1+xy}$

9 **(a)**

$$\because \tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$$
$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right) = \frac{\pi}{2}$$
$$\Rightarrow \frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}} = \tan\frac{\pi}{2} \Rightarrow 1 - \frac{ab}{x^2} = 0$$
$$\Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$$

10 **(d)**

$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = \frac{\pi}{3} + 2.\frac{\pi}{6} = \frac{2\pi}{3}$

11 **(d)**

We have,

$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$

$= 2\tan^{-1}\left(\frac{2/5}{1-1/25}\right) - \tan^{-1}\frac{1}{239}$

$= 2\tan^{-1}(5/12) - \tan^{-1} 1/239$

$= \tan^{-1}\left(\frac{2(2/12)}{(1-5/12)^2}\right) - \tan^{-1}\frac{1}{239}$

$= \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239}$

$= \tan^{-1}\left(\frac{120/119 - 1/239}{1 + 120/119 \times 1/239}\right)$

$= \tan^{-1}\left(\frac{28569}{28569}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

12 **(c)**

Since, $2\sin^{-1} = \sin^{-1}(2x\sqrt{1-x^2})$

Range of right hand side is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow -\frac{\pi}{2} \leq 2\sin^{-1}x \leq \frac{\pi}{2}$$
$$\Rightarrow \frac{\pi}{4} \leq \sin^{-1}x \leq \frac{\pi}{4}$$
$$\Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

14 **(b)**

Sum of two given angles is

$= \cot^{-1}2 + \cot^{-1}3$

$= \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

So, the third angle is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

15 **(a)**

Roots of equation $x^2 - 9x + 8 = 0$ are 1 and 8

Let $y = [\sin^2\alpha + \sin^4\alpha + \sin^6\alpha + \ldots\infty]\log_e 2$

$$\Rightarrow y = \frac{\sin^2\alpha}{1-\sin^2\alpha}\log_e 2 = \tan^2\alpha\log_e 2$$
$$\Rightarrow y = \log_e 2^{\tan^2\alpha}$$
$$\Rightarrow e^y = 2^{\tan^2\alpha}$$

According to question,

$$2^{\tan^2\alpha} = 8 = 2^3 \Rightarrow \tan^2\alpha = 3$$
$$\Rightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$
$$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3} = \alpha$$

16 **(a)**

Let $\cos^{-1}x = \theta$. Then, $x = \cos\theta$

Also, $\frac{1}{2} \leq x \leq 1 \Rightarrow \frac{1}{2} \leq \cos\theta \leq 1 \Rightarrow 0 \leq \theta \leq \frac{\pi}{3}$

Now,

$\cos^{-1}(4x^3 - 3x) = \cos^{-1}(\cos 3\theta)$

$= 3\theta = 3\cos^{-1}x \begin{bmatrix} \because 0 \leq \theta \leq \frac{\pi}{3} \\ \Rightarrow 0 \leq 3\theta \leq \pi \end{bmatrix}$

17 **(b)**

Let $\sin^{-1}x = \theta$. Then, $x = \sin\theta$ and $\sqrt{1-x^2} = \cos\theta$

Now,

$\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= \sin^{-1}(\sin 2\theta) = 2\theta$, if $-\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$

$= 2\sin^{-1}x$, if $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ i.e. if $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

$\therefore \sin^{-1}\left(2x\sqrt{1-x^2}\right) - 2\sin^{-1}x = 0$, if $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

18 **(c)**

$$\because [\sin^{-1}x] > [\cos^{-1}x]$$
$$\Rightarrow x > 0$$

Here, $[\cos^{-1}x] = \begin{cases} 0, & x \in (\cos 1, 1) \\ 1, & x \in (0, \cos 1) \end{cases}$

and, $[\sin^{-1}x] = \begin{cases} 0, & x \in (0, \sin 1) \\ 1, & x \in (\sin 1, 1) \end{cases}$

$$\therefore x \in [\sin 1, 1)$$
$$\because \left[\frac{x}{2}\right] = 1$$

Or we say that $x \in [\sin 1, 1]$

19 **(a)**

We have, $1 \leq \sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1}x \leq \frac{\pi}{2}$

$$\Rightarrow \sin 1 \leq \cos^{-1}\sin^{-1}\tan^{-1}x \leq 1$$
$$\Rightarrow \cos\sin 1 \geq \sin^{-1}\tan^{-1}x \geq \cos 1$$
$$\Rightarrow \sin\cos\sin 1 \geq \tan^{-1}x \geq \sin\cos 1$$
$$\Rightarrow \tan\sin\cos\sin 1 \geq x \geq \tan\sin\cos 1$$
$$\therefore x \in [\tan\sin\cos 1, \tan\sin\cos\sin 1]$$

20 **(d)**

Given, $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$

$$\therefore \quad \tan^{-1}x + 2\tan^{-1}\frac{1}{x} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}x + \tan^{-1}\left(\frac{2\left(\frac{1}{x}\right)}{1 - \left(\frac{1}{x}\right)^2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\left(\frac{2x}{x^2 - 1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{x + \frac{2x}{x^2 - 1}}{1 - \frac{2x^2}{x^2 - 1}}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \qquad \frac{x(x^2 + 1)}{-1(x^2 + 1)} = -\sqrt{3}$$

$$\Rightarrow \qquad x = \sqrt{3}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	D	C	A	C	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	C	B	A	A	B	C	A	D