DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :7

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(b)**

Given expression

$$= \tan\left[\tan^{-1}\frac{a_2-a_1}{1+a_1a_2}+\tan^{-1}\frac{a_3-a_2}{1+a_2a_3}+...+\tan^{-1}\frac{a_n-a_{n-1}}{1+a_na_{n-1}}\right]$$

$$= \tan[\tan^{-1}a_2-\tan^{-1}a_1+\tan^{-1}a_3-\tan^{-1}a_2+...+\tan^{-1}a_n-\tan^{-1}a_{n-1}]$$

$$= \tan[\tan^{-1}a_n-\tan^{-1}a_1]=\frac{a_n-a_1}{1+a_1a_n}$$

$$=\frac{(n-1)d}{1+a_1a_n}$$

2 **(a)**

$$\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right)=\sin\left(\sin^{-1}2\sqrt{\frac{63}{65}}\sqrt{1-\frac{63}{65}}\right)$$

$$=\sin\left(\sin^{-1}\frac{2\sqrt{126}}{65}\right)=\frac{2\sqrt{126}}{65}$$

3 **(b)**

$$\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow\left(\frac{\pi}{2}-\cos^{-1}x\right)-\cos^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow 2\cos^{-1}x=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$$

$$\Rightarrow \cos^{-1}x=\frac{\pi}{6}\Rightarrow x=\frac{\sqrt{3}}{2}$$

4 **(c)**

We know that

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)=2\tan^{-1}x$for all $x\in[-1,1]$

And,

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1} x$for all $x \in [0, \infty)$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 4\tan^{-1} x$for all $x \in [0, 1]$

5 **(c)**

$$\tan^{-1}\left(\frac{c_1 - y}{c_1 y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2 c_1}\right) + \tan^{-1}\left(\frac{c_3 - c_2}{1 + c_3 c_2}\right) + \dots + \tan^{-1}\frac{1}{c_n}$$

$$= \tan^{-1}\left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y}\cdot\frac{1}{c_1}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1 c_2}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2 c_3}}\right) + \dots + \tan^{-1}\frac{1}{c_n}$$

$$= \tan^{-1}\frac{x}{y} - \tan^{-1}\frac{1}{c_1} + \tan^{-1}\frac{1}{c_1} - \tan^{-1}\frac{1}{c_2} + \tan^{-1}\frac{1}{c\_2}$$

$$- \tan^{-1}\frac{1}{c_3} + \dots + \tan^{-1}\frac{1}{c_{n-1}} - \tan^{-1}\frac{1}{c_n} + \tan^{-1}\frac{1}{c_n}$$

$$= \tan^{-1}\left(\frac{x}{y}\right)$$

6 **(c)**

We have, $\tan^{-1} a + \tan^{-1} b = \sin^{-1} 1 - \tan^{-1} c$

$\Rightarrow \tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left\{\frac{a + b + c - abc}{1 - (ab + bc + ca)}\right\} = \frac{\pi}{2} \Rightarrow ab + bc + ca = 1$

7 **(d)**

$\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$

$= \cos\left[\tan^{-1}\left\{\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}\right]$

$= \cos\left[\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right]$

$= \cos\left[\cos^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right]$

$= \sqrt{\frac{1+x^2}{2+x^2}}$

8 **(b)**

$$\because 0 \le \cos^{-1} x \le \pi$$

And $0 < \cot^{-1} x < \pi$

Given, $[\cot^{-1} x] + [\cot^{-1} x] = 0$

$$\Rightarrow [\cot^{-1} x] = 0 \text{ and } [\cos^{-1} x] = 0$$

$$\Rightarrow 0 < \cot^{-1} x < 1 \text{ and } 0 \le \cos^{-1} x < 1$$

$$\therefore x \in (\cot 1, \infty) \text{ and } x \in (\cos 1, 1)$$
$$\Rightarrow x \in (\cot 1, 1)$$

9 **(d)**

Given, $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$

And we know that $0 \le \cos^{-1} x \le \pi$

$\therefore$ We know

$\cos^{-1} x = \pi, \cos^{-1} y = \pi, \cos^{-1} z = \pi$

$\therefore x = y = z = \cos \pi = -1$

$\therefore xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1) = 3$

10 **(c)**

We have,

$\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(1 + x) = \frac{\pi}{2} - \tan^{-1}(1 - x)$

$\Rightarrow \tan^{-1}(1 + x) = \cot^{-1}(1 - x)$

$\Rightarrow \tan^{-1}(1 + x) = \tan^{-1}\left(\frac{1}{1 - x}\right)$

$\Rightarrow 1 + x = \frac{1}{1 - x} \Rightarrow 1 - x^2 = 1 \Rightarrow x = 0$

12 **(a)**

Given equation is

$$2 \cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$$
$$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$$
$$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$$
$$\Rightarrow \cos^{-1} x = \frac{4\pi}{3}$$

Which is not possible as $\cos^{-1} x \in [0, \pi]$.

13 **(a)**

We know that $|\sin^{-1} x| \le \frac{\pi}{2}$

$\therefore \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

$\Rightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$

$\Rightarrow x = y = z = \sin\frac{\pi}{2} = 1$

$\therefore x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}} = 3 - \frac{9}{3} - 0$

14 **(d)**

We have,

$\sin\left(\sin^{-1} 1/5 + \cos^{-1} x\right) = 1$

$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1} x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1} x \Rightarrow \sin^{-1}\frac{1}{5} = \sin^{-1} x \Rightarrow x = \frac{1}{5}$

15 **(d)**

$$\because f(x) = ax + b$$
$$\therefore f'(x) = a > 0$$

$\Rightarrow f(x)$is an increasing function.

$$\therefore f(-1) = 0 \text{ and } f(1) = 2$$

Or $-a + b = 0$

$$\text{and } a + b = 2$$

then, $a = b = 1$

$$\Rightarrow f(x) = x + 1$$

Now, $\cot[\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18]$

$$= \cot\left\{\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{15}{35}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{3}{11}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{1\cdot 1}\cdot\frac{1}{18}}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{65}{195}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{1}{3}\right)\right\}$$

$$= \cot(\cot^{-1} 3) = 3 = 1 + 2 = f(2)$$

16 **(d)**

$$\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$$
$$\Rightarrow 2\tan^{-1} a - 2\tan^{-1} b = 2\tan^{-1} x$$
$$\Rightarrow \tan^{-1}\frac{a-b}{1+ab} = \tan^{-1} x$$
$$\Rightarrow x = \frac{a-b}{1+ab}$$

17 **(c)**

We have,

$\cot^{-1} 3 + \operatorname{cosec}^{-1}\sqrt{5} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2} = \tan^{-1} 1 = \frac{\pi}{4}$

18 **(c)**

$$\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$$

$$= \tan\left[\frac{1}{2}\cdot 2\tan^{-1} a + \frac{1}{2}\cdot 2\tan^{-1} a\right]$$

$$= \tan\,(2\tan^{-1} a)$$

$$= \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$$

$$= \frac{2a}{1-a^2}$$

19 **(a)**

Given, $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$

$\therefore \quad \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$

$\Rightarrow \quad \frac{x+y}{1-xy} = 1$

$\Rightarrow \quad x + y + xy = 1$

20 **(a)**

Let $\cos^{-1} x = \theta$. Then, $x = \cos\theta$

Also, $0 \le x \le 1 \Rightarrow 0 \le \cos\theta \le 1 \Rightarrow 0 \le \theta \le \frac{\pi}{2}$

Now,

$\cos^{-1}(2x^2 - 1) = \cos^{-1}(2\cos^2\theta - 1) = \cos^{-1}(\cos 2\theta)$

$= 2\theta = 2\cos^{-1} x \quad [\because 0 \le 2\theta \le \pi]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	C	C	C	D	B	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	A	D	D	D	C	C	A	A