DPP
DAILY PRACTICE PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :6

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(d)**

We have,

$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\tan^{-1}\left\{\frac{1/2+2/9}{1-1/4\times 2/9}\right\}=\tan^{-1}\left(\frac{1}{2}\right)$

2 **(c)**

Since, $-\frac{\pi}{2}\le \sin^{-1}x\le\frac{\pi}{2}$

$$\therefore \sin^{-1}\alpha=\frac{\pi}{2}, \sin^{-1}\beta=\frac{\pi}{2} \text{ and } \sin^{-1}\gamma=\frac{\pi}{2}$$

$$\therefore \alpha=\beta=\gamma=1$$

Thus ,$\alpha\beta+\alpha\gamma+\gamma\beta=3$

3 **(d)**

We know that,

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$$=\frac{\frac{x\sqrt{3}}{2k-x}-\frac{2x-k}{k\sqrt{3}}}{1+\frac{x\sqrt{3}}{2k-x}\cdot\frac{2x-k}{k\sqrt{3}}}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow A-B=30°$$

4 **(c)**

Given that, $\angle A=\tan^{-1}2, \angle B=\tan^{-1}3$

We know that, $\angle A+\angle B+\angle C=\pi$

$$\Rightarrow \tan^{-1}2+\tan^{-1}3+\angle C=\pi$$

$$\Rightarrow \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right)+\angle C=\pi$$

$$\Rightarrow \tan^{-1}(-1)+\angle C=\pi$$

$$\Rightarrow \frac{3\pi}{4}+\angle C=\pi$$

$$\Rightarrow \angle C=\frac{\pi}{4}$$

5 **(a)**

We have,

$$(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$$

$$\Rightarrow(\tan^{-1}x+\cot^{-1}x)^2-2\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)=\frac{5\pi^2}{8}$$

$$\Rightarrow\frac{\pi^2}{4}-2\times\frac{\pi}{2}\tan^{-1}x+2(\tan^{-1}x)^2=\frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1}x)^2-\pi\tan^{-1}x-\frac{3\pi^2}{8}=0$$

$$\Rightarrow\tan^{-1}x=-\frac{\pi}{6},\frac{3\pi}{4}\Rightarrow\tan^{-1}x=-\frac{\pi}{4}\Rightarrow x=-1$$

6 **(d)**

$$4\tan^{-1}\frac{1}{5}=2\left[2\tan^{-1}\frac{1}{5}\right]$$

$$=2\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}}=2\tan^{-1}\frac{5}{12}$$

$$=\tan^{-1}\frac{\frac{10}{12}}{1-\frac{25}{144}}$$

$$=\tan^{-1}\frac{120}{119}$$

So, $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}=\tan^{-1}\frac{120}{119}-\tan^{-1}\frac{1}{239}$

$$=\tan^{-1}\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\times\frac{1}{239}}$$

$$=\tan^{-1}\frac{(120\times 239)-119}{(119\times 239)+120}$$

$$=\tan^{-1}\frac{28561}{28561}=\tan^{-1}1=\frac{\pi}{4}$$

7 **(b)**

Given, $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$

$$\Rightarrow\left(\frac{\pi}{2}-\cos^{-1}x\right)-\cos^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow\cos^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow x=\frac{\sqrt{3}}{2}$$

8 **(d)**

$$\because f(x)=ax+b$$
$$\therefore f'(x)=a>0$$

$\Rightarrow f(x)$is an increasing function.

$$\therefore f(-1)=0 \text{ and } f(1)=2$$

Or $-a+b=0$

$$\text{and } a+b=2$$

then, $a=b=1$

$$\Rightarrow f(x)=x+1$$

Now, $\cot\left[\cot^{-1}7+\cot^{-1}8+\cot^{-1}18\right]$

$$=\cot\left\{\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$=\cot\left\{\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}}\right)+\tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$=\cot\left\{\tan^{-1}\left(\frac{15}{35}\right)+\tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$=\cot\left\{\tan^{-1}\left(\frac{3}{11}\right)+\tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$=\cot\left\{\tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{1\cdot 1}\cdot\frac{1}{18}}\right)\right\}$$

$$=\cot\left\{\tan^{-1}\left(\frac{65}{195}\right)\right\}$$

$$=\cot\left\{\tan^{-1}\left(\frac{1}{3}\right)\right\}$$

$$=\cot(\cot^{-1}3)=3=1+2=f(2)$$

9 **(d)**

$\cos\left(\frac{33\pi}{5}\right)=\cos\left(6\pi+\frac{3\pi}{5}\right)=\cos\frac{3\pi}{5}$

$=\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)=\sin\left(-\frac{\pi}{10}\right)$

$=\sin^{-1}\sin\left(-\frac{\pi}{10}\right)=-\frac{\pi}{10}$

10 **(c)**

Given equation is

$\cos^{-1}x+\cos^{-1}2x+\pi=0$

$\Rightarrow \quad \cos^{-1}x+\cos^{-1}2x=-\pi$

$\Rightarrow \cos^{-1}\left(x.2x-\sqrt{1-x^2}\sqrt{1-4x^2}\right)=-\pi$

$\Rightarrow\ 2x^2-\sqrt{1-x^2}\sqrt{1-4x^2}=-1$

$\Rightarrow\ (1+2x^2)=\sqrt{1-x^2}\sqrt{1-4x^2}$

On squaring both sides, we get

$$1 + 4x^2 + 4x^2 = (1 - x^2)(1 - 4x^2)$$
$$\Rightarrow\ 1 + 4x^4 + 4x^2 = 1 - 5x^2 + 4x^4$$
$$\Rightarrow\ 9x^2 = 0$$
$$\Rightarrow\ x = 0$$

But $x = 0$ is not satisfied the given equation.

$\therefore$ The number of real solution is zero.

11 **(c)**

Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right) = \alpha$. Then,

$\cos\alpha = \frac{\sqrt{5}}{3}$, where $0 < \alpha < \frac{\pi}{2}$

Now,

$$\tan\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$$

$$\Rightarrow \tan\frac{\alpha}{2} = \sqrt{\frac{1 - \sqrt{5}/3}{1 + \sqrt{5}/3}}$$

$$\Rightarrow \tan\frac{\alpha}{2} = \sqrt{\frac{3 - \sqrt{5}}{3 + \sqrt{5}}} = \sqrt{\frac{(3 - \sqrt{5})^2}{9 - 5}} = \frac{1}{2}(3 - \sqrt{5})$$

$$\therefore \tan\left\{\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\} = \frac{3 - \sqrt{5}}{2}$$

12 **(c)**

$$\sin\left[2\cos^{-1}\frac{\sqrt{5}}{3}\right] = \sin\left[\cos^{-1}\left\{2.\left(\frac{\sqrt{5}}{3}\right)^2 - 1\right\}\right]$$

$$[\because 2\cos^{-1}x = \cos^{-1}(2x^2 - 1)]$$

$$= \sin\left[\cos^{-1}\left(\frac{1}{9}\right)\right]$$

$$= \sin\left[\sin^{-1}\sqrt{1 - \left(\frac{1}{9}\right)^2}\right]$$

$$[\because \cos^{-1}x = \sin^{-1}(\sqrt{1 - x^2})]$$

$$= \frac{4\sqrt{5}}{9}$$

13 **(c)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$$x < -\frac{1}{\sqrt{3}} \Rightarrow \tan\theta < -\frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{2} < \theta < -\frac{\pi}{6}$$

Now,

$\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$= \tan^{-1}(\tan 3\theta)$

$= \tan^{-1}(\tan(\pi+3\theta)) = \pi + 3\theta = \pi + 3\tan^{-1}x$

14 **(c)**

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$
$$= \frac{\pi}{3} + 2\cdot\frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

15 **(d)**

Given, $\sin\left[\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}x\right] = 1$

$\therefore \quad \sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \qquad \sin^{-1}\left(\frac{1}{5}\right) = \sin^{-1}x$

$\Rightarrow \qquad x = \frac{1}{5}$

16 **(c)**

Given that ,$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$$\because 0 \le \cos^{-1}x \le \pi$$

Similarly, $0 \le \cos^{-1}y \le \pi$

And $0 \le \cos^{-1}z \le \pi$

Here, $\cos^{-1}x\cos^{-1}y = \cos^{-1}z = \pi$

$$\Rightarrow x = y = z = \cos\pi = -1$$
$$\therefore xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1)$$

=1+1+1=3

17 **(a)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$0 \le x \le \infty \Rightarrow 0 \le \theta < \frac{\pi}{2} \Rightarrow 0 \le 2\theta < \pi$

Now,

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}(\cos 2\theta)$

$= 2\theta \left[\because 0 \le \theta < \frac{\pi}{2} \Rightarrow 0 \le 2\theta < \pi\right]$

$= 2\tan^{-1}x$

18 **(d)**

$\cos[2\tan^{-1}(-7)] = \cos\left[\cos^{-1}\left(\frac{1-49}{1+49}\right)\right]$

$\qquad = \cos\left[\pi - \cos^{-1}\left(\frac{48}{50}\right)\right]$

$$= -\cos\cos^{-1}\left(\frac{48}{50}\right)$$

$$= -\frac{24}{25}$$

19 **(d)**

We have,

$\sin\left(4\tan^{-1}\frac{1}{3}\right)$

$= 2\sin\left(2\tan^{-1}\frac{1}{3}\right)\cos\left(2\tan^{-1}\frac{1}{3}\right)$

$= 2\sin\left(\tan^{-1}\frac{3}{4}\right)\cos\left(\tan^{-1}\frac{3}{4}\right)$

$= 2\sin\left(\sin^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}\frac{4}{5}\right) = 2\times\frac{3}{5}\times\frac{4}{5} = \frac{24}{25}$

And,

$\cos\left(2\tan^{-1}\frac{1}{7}\right) = \cos\left(\tan^{-1}\frac{7}{24}\right) = \cos\left(\cos^{-1}\frac{24}{25}\right) = \frac{24}{25}$

Hence, the value of given expression is 0

20 **(c)**

Given that ,$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$$\because 0 \le \cos^{-1}x \le \pi$$

Similarly, $0 \le \cos^{-1}y \le \pi$

And $0 \le \cos^{-1}z \le \pi$

Here, $\cos^{-1}x\cos^{-1}y = \cos^{-1}z = \pi$

$$\Rightarrow x = y = z = \cos\pi = -1$$

$$\therefore xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1)$$

=1+1+1=3

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	D	C	A	D	B	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	C	C	D	C	A	D	D	C