DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-INVERSE TRIGONOMETRICE FUNCTIONS

1 **(c)**

Since, $\tan^{-1} x$ and $\cot^{-1} x$ exists for all $x \in$ R and $\cos^{-1}(2-x)$ exists, if $-1 \leq 2-x \leq 1$

$$\therefore \tan^{-1} x - \cot^{-1} x = \cos^{-1}(2-x)$$

Is possible only if $1 \leq x \leq 3$.

Thus the solution of given equation is [1, 3].

2 **(a)**

Since, $0 \leq \cos^{-1}\left(\frac{x^2}{2} + \sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right) \leq \frac{\pi}{2}$

Because $\cos^{-1} x$ is in first quadrant when x is positive

And $\cos^{-1}\frac{x}{2} - \cos^{-1} x \geq 0$

So, $\cos^{-1}\frac{x}{2} \geq \cos^{-1} x$

Also, $\left|\frac{x}{2}\right| \leq 1, |x| \leq 1 \Rightarrow |x| \leq 1$

3 **(b)**

We have,

$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \tan^{-1}\left\{\frac{2x}{1-(x^2-1)}\right\} = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$

$\Rightarrow 8x^2 + 62x - 16 = 0 \Rightarrow (4x-1)(x+8) = 0 \Rightarrow x = \frac{1}{4}, -8$

4 **(b)**

$$3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

On putting $x = \tan\theta$, we get

$$3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$$

$$\Rightarrow 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$

$$\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3}$$
$$\Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$$
$$\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \tan^{-1} x = \frac{\pi}{6}$$
$$\Rightarrow x = \tan\frac{\pi}{6} \Rightarrow x = \frac{1}{\sqrt{3}}$$

5 **(b)**

Given that, $x^2 + y^2 + z^2 = r^2$

Now, $\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xe}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$

$$= \tan^{-1}\left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \left(\frac{x^2 + y^2 + z^2}{r^2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \frac{r^2}{r^2}}\right]$$

$$= \tan^{-2}\infty = \frac{\pi}{2}$$

6 **(d)**

We have, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$

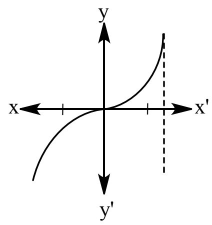
$$= (\sin^{-1} + \cos^{-1} x)^3 - 3\sin^{-1} x \cos^{-1} x\,(\sin^{-1} x + \cos^{-1} x)$$
$$= \frac{\pi^3}{8} - 3(\sin^{-1} x \cos^{-1} x)\frac{\pi}{2}$$
$$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)$$
$$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x + \frac{3\pi}{2}(\sin^{-1} x)^2$$
$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x\right]$$
$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right]$$
$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32}$$
$$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1} x - \frac{\pi}{4}\right)^2$$

∴ The least value is $\frac{\pi^3}{32}$

Since, $\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \le \left(\frac{3\pi}{4}\right)^2$

∴ The greatest value is $\frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$

7 **(d)**



Hence, the line $x = 1$ is a tangent to the function.

8 **(c)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$ and $\sqrt{1 - x^2} = \cos\theta$

Now,

$$-1 \leq x \leq -\frac{1}{\sqrt{2}} \Rightarrow -1 \leq \sin\theta \leq -\frac{1}{\sqrt{2}} \Rightarrow -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{4}$$

$$\therefore \sin^{-1}\left(2x\sqrt{1 - x^2}\right) = \sin^{-1}(\sin 2\theta) = \sin^{-1}(-\sin(\pi + 2\theta))$$

$$= \sin^{-1}(\sin(-\pi - 2\theta))$$

$$= -\pi - 2\theta \left[\because -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{4} \Rightarrow -\frac{\pi}{2} \leq -\pi - 2\theta \leq 0\right]$$

$$= -\pi - 2\sin^{-1} x$$

9 **(a)**

$$\theta = \tan^{-1}\sqrt{\frac{a(a + b + c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a + b + c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a + b + c)}{ab}}$$

Let $s^2 = \frac{a + b + c}{abc}$

Hence, $\theta = \tan^{-1}\sqrt{a^2 s^2} + \tan^{-1}\sqrt{b^2 s^2} + \tan^{-1}\sqrt{c^2 s^2}$

$$= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs)$$

$$= \tan^{-1}\left[\frac{as + bs + cs - abcs^3}{1 - abs^2 - acs^2 - bcs^2}\right]$$

Hence, $\tan\theta = \left[\frac{s[a + b + c) - abcs^2]}{1 - (ab + bc + ca)s^2}\right]$

$$= \left[\frac{s[(a + b + c) - (a + b + c)]}{1 - s^2(ab + bc + ca)}\right] = 0$$

10 **(a)**

We have,

$\theta \in [4\pi, 5\pi] \Rightarrow -4\pi + \theta \in [0, \pi]$

Also,

$\cos(-4\pi + \theta) = \cos(4\pi - \theta) = \cos\theta$

$\therefore \cos^{-1}(\cos\theta) = \cos^{-1}\{\cos(-4\pi + \theta)\} = -4\pi + \theta$

11 **(c)**

Given, $\sin^{-1} x = 2\sin^{-1} a$

Since, $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{2} \leq 2\sin^{-1} a \leq \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4} \leq \sin^{-1} a \leq \frac{\pi}{4}$

$\Rightarrow \quad \sin\left(-\frac{\pi}{4}\right) \leq a \leq \sin\frac{\pi}{4}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

$\therefore \quad |a| \leq \frac{1}{\sqrt{2}}$

12 **(b)**

We have,

$\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

$\Rightarrow \sin^{-1} 2x = \frac{\pi}{3} - \sin^{-1} x$

$\Rightarrow 2x = \sin\left(\frac{\pi}{3} - \sin^{-1} x\right)$

$\Rightarrow 2x = \frac{\sqrt{3}}{2}\cos(\sin^{-1} x) - \frac{1}{2}\sin(\sin^{-1} x)$

$\Rightarrow 2x = \frac{\sqrt{3}}{2} \times \sqrt{1-x^2} - \frac{x}{2}$

$\Rightarrow \frac{5x}{2} = \frac{\sqrt{3}}{2}\sqrt{1-x^2}$

$\Rightarrow 25x^2 = 3 - 3x^2$

$\Rightarrow x = \pm\frac{1}{2}\sqrt{\frac{3}{7}} \Rightarrow x = \frac{1}{2}\sqrt{\frac{3}{7}} \qquad [\because \text{RHS} > 0 \quad \therefore x > 0]$

13 **(c)**

Since, $2\sin^{-1} = \sin^{-1}(2x\sqrt{1-x^2})$

Range of right hand side is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow -\frac{\pi}{2} \leq 2\sin^{-1} x \leq \frac{\pi}{2}$$
$$\Rightarrow \frac{\pi}{4} \leq \sin^{-1} x \leq \frac{\pi}{4}$$
$$\Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

14 **(c)**

$$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$$
$$\Rightarrow \cos^{-1}\left(xy - \sqrt{1-y^2}\sqrt{1-x^2}\right) = \pi - \cos^{-1} z$$
$$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos(\pi - \cos^{-1} z)$$
$$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$$
$$\Rightarrow xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$$

On squaring both sides, we get

$$x^2y^2 + z^2 + 2xyz - 1 - x^2 - y^2 + x^2y^2$$
$$\Rightarrow x^2 + y^2 + z^2 = 1 - 2xyz$$

15 **(b)**

$\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right) = \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

$$= \cot \tan^{-1}\left[\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{1}{2}}\right]$$

$$= \cot\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$$

$$= \frac{6}{17}$$

16 **(b)**

We have, $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$

or$x\sqrt{1-y^2} + y\sqrt{1-x^2 = z}$

or$x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{(1-x^2)}$

or$(x^2 - z^2 - y^2)^2 = 4y^2z^2(1-x^2)$

or$x^4 + y^4 + z^4 - 2x^2z^2 + 2y^2z^2 - 2x^2y^2 + 4x^2y^2z^2 - 4y^2z^2 = 0$

or$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

$\therefore\ k = 2$

17 **(b)**

Given, $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$

$\Rightarrow \quad \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = \pi$

$\Rightarrow \quad \frac{x+y+z-xyz}{1-xy-yz-zx} = 0$

$\Rightarrow \quad x+y+z = xyz$

18 **(b)**

Since, 1 radian $= \frac{7\pi}{22}$

$\therefore\ 12 \text{ radian} = \frac{7\pi}{22} \times 12 = \frac{42\pi}{11} = 4\pi - \frac{2\pi}{11}$

And 14 radian $= \frac{7\pi}{22} \times 14 = \frac{49\pi}{11}$

$= 4\pi + \frac{5\pi}{11}$

$\therefore\ \cos^{-1}(\cos 12) - \sin^{-1}(\sin 14)$

$$= \cos^{-1}\cos\left(4\pi - \frac{2\pi}{11}\right) - \sin^{-1}\left[\sin\left(4\pi + \frac{5\pi}{11}\right)\right]$$

$$= \cos^{-1}\cos\left(\frac{2\pi}{11}\right) - \sin^{-1}\left(\sin\frac{5\pi}{11}\right)$$

$$= 4\pi - 12 - (14 - 4\pi) = 8\pi - 26$$

19 **(b)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$

Also,

$\frac{1}{2} \le x \le 1 \Rightarrow \frac{1}{2} \le \sin\theta \le 1 \Rightarrow \frac{\pi}{6} \le \theta \le \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \le 3\theta \le \frac{3\pi}{2}$

Now,

$\sin^{-1}(3x - 4x^3)$
$= \sin^{-1}(\sin 3\theta)$
$= \sin^{-1}(\sin(\pi - 3\theta))$
$= \pi - 3\theta \left[\because \frac{\pi}{2} \leq 3\theta \leq \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} \leq \pi - 3\theta \leq \frac{\pi}{2}\right]$
$= \pi - 3\sin^{-1} x$

20 **(c)**

$\cos(4095°) = \cos(45 \times 90° + 45°)$
$= -\sin 45°$
$= -\sin\frac{\pi}{4}$
$= \sin\left(-\frac{\pi}{4}\right)$

$\therefore \sin^{-1}\{\cos(4095°)\}$
$= \sin^{-1}\sin\left(-\frac{\pi}{4}\right)$
$= -\frac{\pi}{4}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	B	B	B	D	D	C	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	C	B	B	B	B	B	C